CW laser light condensation

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Abstract: We present a first experimental demonstration of classical CW laser light condensation (LC) in the frequency (mode) domain that verifies its prediction (Fischer and Weill, <jrn>Opt. Express **20**, 26704 (2012)</jrn>). LC is based on weighting the modes in a noisy environment in a loss-gain measure compared to an energy (frequency) scale in Bose-Einstein condensation (BEC). It is characterized by a sharp transition from multi- to single-mode oscillation, occurring when the spectral-filtering (loss-trap) has near the lowest-loss mode ("ground-state") a power-law dependence with an exponent smaller than 1. An important meaning of the many-mode LC system stems from its relation to lasing and photon-BEC.

OCIS codes: (270.3430) Laser theory; (020.1475) Bose-Einstein condensates.

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#254555 Received 24 Nov 2015; revised 29 Feb 2016; accepted 3 Mar 2016; published 15 Mar 2016

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1. Introduction

Light spectral transition from multi to single frequency (or multi to single mode) oscillation in optical cavities can occur in lasers or maybe via more exotic mechanisms such as classical and quantum based photon Bose-Einstein condensation. CW lasers are naturally an important experimental platform for following, understanding and realizing these scenarios. Bose Einstein condensation (BEC) is a special many non-interacting boson phenomenon that was observed in atomic particles at ultra-low temperatures [1–3]. In recent years there were reports on experimental observation of BEC in non-atomic particles such as polaritons [4,5], magnons [6], and photons [7]. Light can also show classical condensation phenomena. We have reported on three classical light condensation (LC) effects in lasers that we describe below [8–11]. In this paper we report on a first experimental demonstration of CW laser light condensation (LC) in the frequency (mode) domain. The experiment verifies and follows the theoretical prediction for this condensation effect [10]. We also add that there were a few other important works on classical light condensation phenomena, such as in nonlinear waves and disordered lasers [12–16].

Our work on classical laser light condensation is a part of a broader study of many-mode lasers as a many body system using powerful methods and tools from statistical mechanics. Noise takes the role of temperature and quantities like entropy, free energy and partition function are essential to explain the basic properties of mode-locked lasers. This approach explained fundamental properties, such as the threshold needed for passive mode-locking. It was shown that mode-locking is nothing but a first order phase transition [17,18]. Other thermodynamic-like aspects were predicted and demonstrated, such as critical phenomena in passive mode-locking [19,20] and that the mode system in active mode-locking is a rare physical realization of the exactly soluble spherical model for magnetic spins in statistical-mechanics [21,22].

2. Classical laser light condensation (LC)

LC is based on "energy" levels measured in a loss-gain scale, where the "ground-state" is the lowest-loss frequency (mode) and noise has the role of temperature [8–11]. The loss scale gives a spectral power occupation hierarchy with an effective chemical potential and a route to laser light-condensation (LC), formally similar to BEC in a potential-well [23], but classical. As said above, LC occurs in single- or multi-pulse actively mode-locked lasers and in many-mode CW lasers with noise without explicit nonlinearity or mode interaction, besides a global constraint on the overall pulse or mode power, like the overall non-interacting particles number in BEC. We presented in the last years the following three classical laser light condensation (LC) systems:

i. The first LC system [8,9] is based on a single pulse in a cavity of an actively mode-locked (AML) laser. Here the "energy" levels are the pulse eigen-functions of the AML lasers with a special modulation. In the common harmonic modulation case they are Hermite-Gaussian functions, where the lowest one is the "ground state" that corresponds to the optimal and shortest pulse [8,9]. The condensation is seen in the cavity time domain or its longitudinal axis.

ii. The second LC was done in a many pulse system in a high harmonic AML laser where the pulses and their power are the levels and "particles" [11].

iii. The third LC system with a many-mode CW laser was theoretically presented a while ago [10] and its first experimental verification is given in this paper. Here the modes and their power are the levels and "particles", and the condensation occurs in the modes spectral domain at the lowest-loss mode.

#254555 Received 24 Nov 2015; revised 29 Feb 2016; accepted 3 Mar 2016; published 15 Mar 2016

In all these LC systems the hierarchy is provided by a loss-well (loss-trap), like the potential-well in BEC needed for condensation in two or lower dimensions [23]. Condensation occurs in the single pulse case for a loss function with a power-law dependence on time (or cavity axis) near the low-loss frequency point when its exponent η is smaller than 2 ($\eta < 2$) [8,9]. In the many-pulse systems, condensation occurs in a loss-trap provided by an envelope modulation with a power-law dependence on time (cavity axis) near the low-loss pulse with an exponent $\eta < 1$) [11]. In the many-mode CW laser the prediction for the spectral loss filtering exponent was also $\eta < 1$ [10]. Experimentally, we demonstrated only the single- and multi-pulse systems [9,11], but not the multi-mode LC [10]. In the present work we give its first experimental demonstration. It is the closest one to lasing and photon-BEC, where condensation occurs in the frequency domain. It is therefore important to realize and follow these different single-frequency oscillation effects since they all occur in laser cavities with gain media. The present experiment sheds light on the important discussion on light and photon condensation in optical cavities [10,17].

3. A summary of the mathematical base of LC in the many-mode system [10]

We briefly summarize the mathematical base of the many-mode classical condensation effect in CW lasers [10]. The formalism is similar to the pulse system LC [11]. The modes and their power serve as the levels and the "particles". The equation of motion for a light mode a_m circulating in the cavity is given by simple multivariate Langevin equations (with shared and variate-dependent coefficients, which is a key part for condensation) [10]:

$$da_m / d\tau = (g - \varepsilon_m) a_m + \Gamma_m \ , \tag{1}$$

where $a_m(\tau)$ is the slowly varying electric field complex amplitude of the mode *m*, τ is the long term time variable that counts cavity roundtrip frames, and g is a slow saturable gain factor, shared by all modes. It functions as a Lagrange multiplier (and $\mu \equiv g - \varepsilon_0$ as "chemical potential") for setting the overall cavity power *P*, and ε_m is spectral filtering (loss dispersion) due to frequency and mode dependent loss, absorption and gain. Γ_m is an additive noise term that can originate from spontaneous emission, or any other internal or external source. It is modeled by a white Gaussian process with covariance $2T$: $<\Gamma_m(\tau)\Gamma_n^*(\tau')> = 2T\,\delta(\tau - \tau')\,\delta_{mn}$, and $<\Gamma_m(\tau)> = 0$, where $< >$ denotes average. We describe the modes frequency by $\Omega = \omega - \omega_0 = (c/n)(|\bar{k}| - k_0)$ (where $\bar{k}$ is the wavevector and *n* is the refractive index), measured with respect to the linewidth center $\omega_0 = (ck_0 / n)$. The modes discrete relative frequencies are Ω_m. With the continuous variable we have $a_m(\tau) \to a(\Omega, \tau)$, and $\varepsilon_m \to \varepsilon(\Omega)$. We note that our study is applicable to discrete modes, as well as to continuous frequencies.

The overall modes power in the cavity obtained from Eq. (1) is:

$$P = \sum_{m=-N}^{N} p_m = \sum_{m=-N}^{N} \frac{T}{\varepsilon_m - g} \ \rightarrow \ P = \frac{T}{\varepsilon_0 - g} + T \int_0^{\varepsilon_N} \frac{\rho(\varepsilon) d\varepsilon}{\varepsilon + \varepsilon_0 - g} \ , \tag{2}$$

where $p_m = < a_m a^*_m >$ is the mean m_{th} mode power. For the integral with the continuous loss variable ε, measured relatively to the lowest mode loss ε_0, we defined the density of loss states (DOS) $\rho(\varepsilon)$ that has a prime role for condensation occurrence. ε_N is the highest mode loss in the cavity, the loss-trap depth.

The first term at the right hand side of Eq. (2) gives the power of the lowest loss (ε_0) mode, and the second term is the power in all of the higher modes. We can notice the

#254555 Received 24 Nov 2015; revised 29 Feb 2016; accepted 3 Mar 2016; published 15 Mar 2016

resemblance to BEC in a potential-well [23]. The weight for each spectral component depends on $\rho(\varepsilon)$ and a distribution factor $T/(\varepsilon+\varepsilon_0-g)$ that replaces the Bose-Einstein statistics. The upper limit of the integral, ε_N, is set by the spectral filtering (loss "potential-well") depth. However, it is the density of the low loss-levels at $\varepsilon \approx 0$ that determines the integral convergence. We therefore take the spectral filtering functional dependence as a power law around ε_0: $\varepsilon = \gamma|\Omega|^{\eta}$. It can be shown that it gives the following density of states (modes) $\rho(\varepsilon) \propto \varepsilon^{\xi}$, where $\xi = \eta^{-1} - 1$ [10]. As P is raised or the noise T is lowered the distribution $p(\varepsilon)$ yields a gradual concentration of the power at lower-loss mode. Nevertheless, when condensation takes place, p_0 gets a macroscopic part of the power ($p_0 >> P/N$ for a large N). It occurs when the integral in Eq. (2) converges at $\varepsilon = 0$, that happens when $\xi > 0$ or $\eta < 1$. Then, the power population of the light at higher than $\varepsilon = 0$ levels stays unchanged at $P_c = T\int_0^{\varepsilon_N} \rho(\varepsilon) d\varepsilon/\varepsilon$, and additional pumping that increases P beyond P_c (or lowering T below T_c) will cause population of the lowest level power p_0 which starts to grow macroscopically. At condensation ($P \geq P_c$, or $T \leq T_c$), the net gain for the lowest-loss mode (the "chemical potential") becomes $\mu \equiv g - \varepsilon_0 = 0$. We therefore have a many mode system with a condensation route that is mathematically similar to BEC, but classical, that is also called Rayleigh-Jeans condensation [16]. As in BEC, there is no direct mode ("particle") interaction, but a global constraint on the overall modes power ("particle number"). Nevertheless, the "energy" hierarchy is in a loss-gain scale, where the "ground-state" is the lowest-loss mode, and the noise has the role of temperature.

4. Experiment and results

The experiment was done in a ring erbium-doped fiber laser (EDFL) with a length of *~20m,* schematically shown in Fig. 1. We had in the system two special parts. The first one provided an external noise injection into the cavity by a controlled ASE source, that, as mentioned above, has the role of "temperature". The second was a spectral filtering element that formed the loss-trap. It was obtained by an intra-cavity controllable wave-shaper that can synthesize and insert frequency dependent loss functions on the spectrum, in our experiment a power-law dependence with an exponent η. The spectral loss filter resolution was ~7*GHz*, compared to the cavity longitudinal mode spacing in our experiment of ~10*MHz,* meaning that the resolution in this experiment was of about 7×10^2 modes, in an overall number of $\sim 2 \times 10^7$ modes/*nm* in the cavity. Therefore, the lowest-loss frequency condensation level included $\sim 7 \times 10^2$ indistinguishable modes measured in p_0. To take the limited resolution into account in our theory, we used a simple numerical simulation to solve Eq. (2) in its discrete form. The loss function $\varepsilon = \gamma|\Omega|^{\eta}$ that determines the values of ε_m was smeared with a Gaussian filter with a 7*GHz* width. Then, the mode spectrum and the total power were calculated for many values of g. The finite resolution of the spectrum analyzer was also taken into account in the figures with the numerically calculated spectra. Obtaining a few mode resolution that matches the wave-shaper can be done by using an analyzer with higher resolution or much shorter cavities which have a larger mode-spacing, but was difficult to implement in our experiment.

The measured and theoretical spectra are shown in Fig. 2 for $\eta = 1/2, 2$ and various power (pumping) levels, while keeping the noise level T ("temperature") unchanged. For $\eta = 1/2$ the spectra become very narrow even at low pumping levels and show condensation. For $\eta = 2$ they are much broader and only strong pumping provides "lasing" lines at several wavelengths near the centeral mode, resulting from nonuniformities in the gain-loss curve, but not condensation. We can see a very good fit between the measurments and the theory. We also varied the noise strength T ("temperature") instead of the light power $P,$ since the dependence is on T/P [8–11].

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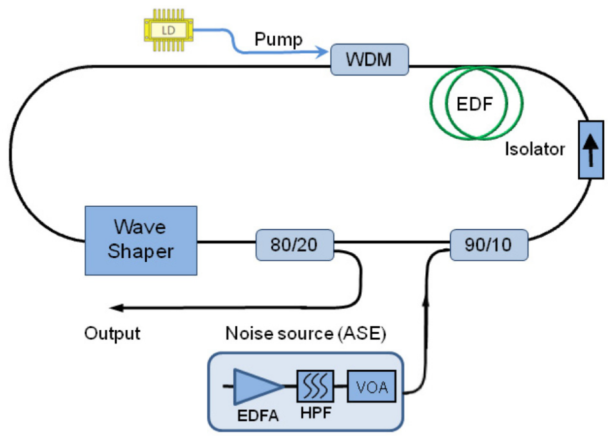

Fig. 1. Scheme of the CW many-mode EDF laser system with two special parts: External noise injection ("temperature") into the cavity by a controlled ASE source, and a wave-shaper that forms the spectral loss-trap.

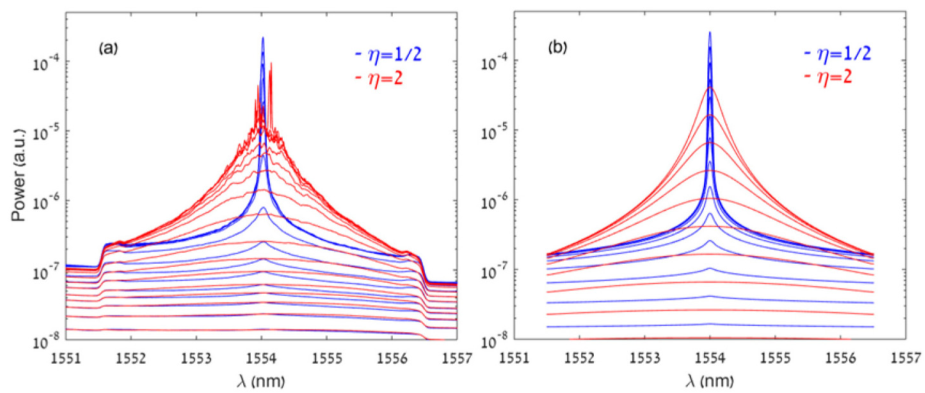

Fig. 2. (a) Experimental and (b) theoretical spectra for $\eta = 1/2, 2$ and various power (pumping) levels. For $\eta = 1/2$ (blue curves) the spectrum becomes very narrow even at low pumping levels and shows condensation. For $\eta = 2$ (red curves) the apectra are broader and only strong pumping provides "lasing" lines at several wavelengths near the centeral mode.

Figure 3(a) shows theoretical and experimental condensation fractions p_0 - power at the lowest-loss modes/wavelength ("ground state") as a function of the total power P for the exponents $\eta = 1/2, 2$. The total intracavity light power in the experiments reached ~3 mW and 20% of it emitted through the output coupler. We can see again dthe condensation effect for $\eta = 1/2$ that starts at $P_c \approx 0.6mW$ and a very small and gradual population fraction for $\eta = 2$. The condensation transition is not sharp due the finite size of the mode system. We added in the figure the large system (thermodynamic limit) graph where the transition is sharp. We also show in Fig. 3(b) the condensation fraction as a function of the noise strength T ("temperature") and a constant power P_e, instead of varying P, since the dependence is on T/P [8–11]. It is done by controlling the optical noise injection into the cavity using amplified spontaneous-emission (ASE) from an erbium-doped fiber (EDF) amplifier. The figure gives the experimental and theoretical lowest modes ("ground state") power for $\eta = 1/2, 2$. The temperature needed for condensation at $\eta = 1/2$ is finite, while for $\eta = 2$ it is near zero. In the experiment it is slightly higher due to the "lasing lines". Again, the condensation transition is not sharp due the finite size of the mode system. We note that there is in the experiment a relatively small but nonzero noise level inside the laser without injection while in the theory T starts from 0, and we therefore included an adjusting shift in T. We also note that the power in this experiment is kept by a control loop almost constant, within 5% changes, except in the low noise region where lasing started to appear.

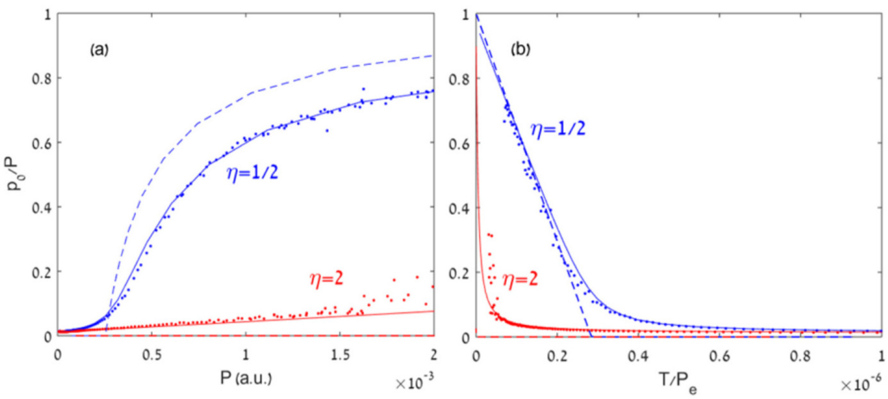

Fig. 3. Experimental (dots) and theoretical (solid lines) condensation fractions as a function of (a) the total power P, with a constant noise $T \approx 0.015mW$, and (b) the noise T ("temperature") normalized by the constant power $P_e \approx 0.625mW$ in the experiment, for two spectral filtering exponents $\eta = 1/2, 2$ (blue and red curve respectively). We can see the condensation transition effect for $\eta = 1/2$ and a very small and gradual population fraction for $\eta = 2$. The power or temperature needed for condensation at $\eta = 1/2$ is finite, while for $\eta = 2$ it is near zero. The transition is not sharp due the finite size of the mode system. The large system (thermodynamic limit) theoretical graph is shown in the dashed lines.

5. Conclusion

The present work is a first experimental demonstration of classical CW laser condensation (LC) in the mode frequency domain. Its importance stems from the similarity to lasing and photon-BEC in optical cavities, as we can have in all of them a transition from multi- to single frequency oscillation that has a strong effect on the coherence. Photon-BEC in the dye-filled microcavity experiment was associated with a spectrum collapse to a single frequency at the lowest transverse mode spectral band, when the power (the photons number) was increased beyond a critical value [7]. Such scenario happens in the present experiment and can be seen in lasers. Except for the difference in the statistics, quantum or classical noise based, an important difference is the condensation state. In the classical LC condensation effects the "energy" measure of photons in multi-mode laser cavities is governed by a loss-gain scale that gives the hierarchy and distribution rather than by photon frequency (energy) scale in thermal equilibrium and BEC. Therefore, the condensation in the quantum case is in the lowest energy or frequency level, while in the classical cases it is in the lowest loss state. This and other points [10,17] can add to the understanding of experimental observations of light mode and frequency collapse in optical cavities.

Acknowledgments

This research was supported by the Pazy Foundation and the Israel Science Foundation (ISF).

#254555 Received 24 Nov 2015; revised 29 Feb 2016; accepted 3 Mar 2016; published 15 Mar 2016