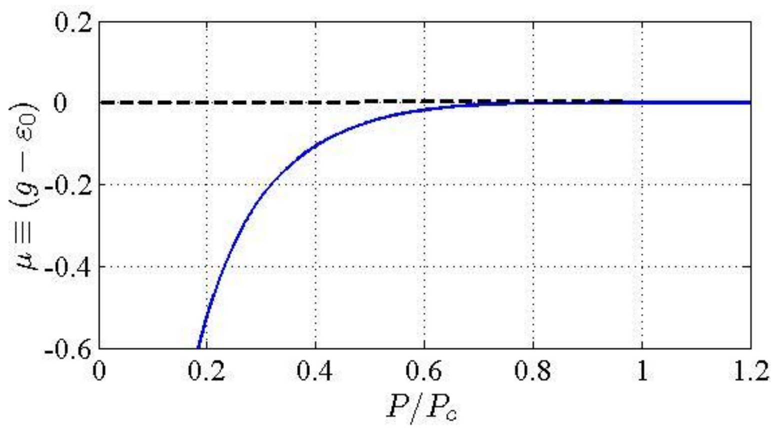

Fig. 3. Normalized "chemical potential" $\mu = (g - \varepsilon_0)$ as a function of the overall normalized laser power P / P_C, for $\tilde{\gamma} = 1$, $\eta = 0.8$, and $\varepsilon_N = 1$.

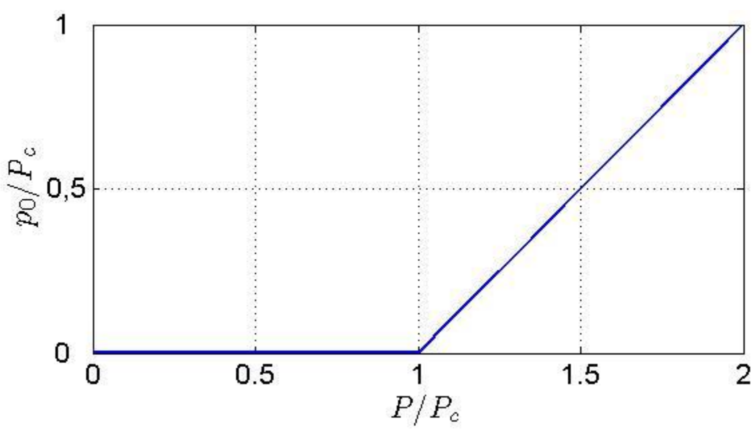

Fig. 4. Condensation transition: Normalized lowest mode power p_0 / P_C as a function of the overall normalized laser power for for $\tilde{\gamma} = 1$, $\eta = 0.8$, and $\varepsilon_N = 1$. Above the phase transition ($P \geq P_C$) all the excess power goes to p_0 - the condensation state.

3. Discussion on lasing, LC, photon-BEC and experiments

LC is formally similar to BEC in having:

a. Non-interacting particles - bosons in BEC and light-modes in LC.

b. Particle distribution - Bose-Einstein statistics vs. loss-dependent mode weighting in LC.

c. A global constraint on the overall bosonic particles number in BEC vs. the overall light-modes power in LC.

d. The chemical potential in BEC corresponds to the lowest-mode gain minus loss in LC. In both cases it is negative below condensation, and approaches and becomes zero at and above the transition.

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e. Certain conditions on the density of states function for bosonic particle and light-modes.

f. A similar mathematical route to condensation in BEC and LC.

There are however a few important differences between LC and BEC, besides the basic one that LC is classical and BEC is a quantum-thermal effect. We specify the main ones:

a. The states (modes) hierarchy ("energy ruler") in LC is measured by loss-gain, compared to photon energy (frequency) in BEC.

b. It is a general noisy system in LC vs. thermal equilibrium, characterized by temperature, in BEC.

c. Laser LC occurs at the lowest-loss state ($\Omega = 0$), which can be anywhere in the spectral band, and not necessarily at the lowest photon energy $(\hbar\omega)$ state, as it is in BEC.

d. The spectrum of the oscillating modes in LC (Fig. 2) is dictated by the loss scale. It is "noisization" rather than the BEC thermalization spectrum [4]. Unlike BEC, the spectra can span over frequencies above, or below, or at both sides of $\Omega = 0$. We also note that in LC, below P_c, there is a hole in $p(\varepsilon)$ near $\Omega = 0$, ($|\bar{k}| = k_0$), resulting from $\rho(\Omega = 0) = 0$, but not in $p(\Omega)$ (in the frequency domain), as seen in Fig. 2. In BEC, prior to condensation, there is a hole in the energy distribution at zero energy (as in black-body radiation spectra), which is always at the lowest possible energy (frequency) [20].

e. Unlike regular BEC [20], there is no dimensional dependence in LC when the condensation results from spectral filtering, that operates in one dimension (Ω, at the $|\bar{k}| \approx k_0$ sphere shell), except for a factor that results from the d-dimensional sphere surface. Nevertheless, when there are in the system other space (or time) dependent loss-gain filtering (or modulation), like in two dimensional transverse mode systems [3,4], the condition for condensation can be different [8].

We elaborate on the LC and BEC connection and the relation to lasers and experiments. Both cases are characterized by a transition to a single-frequency oscillation. Such a transition can also occur in regular lasers, like in ideally homogeneous lasers [23]. In a way, every single-frequency laser could have been regarded as a condensate state. (Photon densities in single-frequency lasers can be much higher than in the microcavity experiment [3].) In those lasers the single-mode oscillation is explained by a nonlinear saturation effect or a simple ad hoc assumption that the first mode to reach threshold clamps the gain profile and therefore eliminates the oscillation of all other modes [23]. In LC, it is embedded in the density of states where high mode states populated by the noise become filled (for certain quasi-continuous mode densities $\rho(\varepsilon)$ with exponents $\eta < 1$), leading to condensation, without an explicit nonlinearity, but with a global constraint, in a similar way to BEC. It is likewise characterized by a negative "chemical potential" $\mu \equiv g - \varepsilon_0$ that becomes zero at condensation (as shown in Fig. 3.) It means that we have a different route to a single-mode oscillation than regular lasers.

In experiments it can be very challenging to relate and identify the different single-frequency oscillation effects, since they are all done in laser cavities. It is possible that experiments in laser cavities fall in LC or a single-mode lasing category, rather than being a thermal photon-BEC phenomenon [3]. We have shown the role of spectral-filtering, inevitable in lasers, that governs the photon gas "climate", the "equilibrium" and the spectra in a pumped many-mode cavity with gain. In the dye-filled microcavity experiment [3,4], it includes the transverse-modes loss-hierarchy (mode-filtering) due to the difference (even if very weak) in their transverse waveform, mirror-reflectivity, gain, absorption and other losses. The

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frequency cutoffs due to far apart longitudinal modes in the microcavity can give the needed abrupt spectral filtering profile. However, even without spectral filtering, LC can occur in the lateral domain. The situation can be similar to what we suggested in Ref. [8] (see there the last paragraph before conclusion), where lateral modulation or confinement, and transverse mode filtering, even if the modes had the same frequency, can cause transversal condensation [8]. Then the condition on the exponent of the lateral loss profile ("potential-well") is very much relaxed. In two-dimensions, any finite positive η near $\Omega = 0$ gives condensation.

There are also other issues, discussed elsewhere and briefly given here, that need consideration in the microcavity experiments [3]. The massively lost photon replacement in the cavity by pumping occurs via spectral-loss criterion. It is generally the case in lasers, being non-Hamiltonian with frequency dependent dissipation, not in thermal equilibrium, and not a grand-canonical ensemble. The microcavity is not an exception. The seemingly large photon "keepers" (the mirrors) do not provide better performance than what we usually have in many other lasers. The microcavity mirrors have ultra high reflectivity of 0.999985 [3,4], and therefore provide a very high finesse. Nevertheless, since the cavity length is small (~1.5μm), important parameters value stay in the usual regime. The photon loss through the mirrors by itself gives a cavity photon lifetime of ~0.3*ns*, that is similar or shorter than usual values in lasers. Likewise, the gain coefficient needed for steady state oscillation is relatively large (~0.2/*cm*). When adding other losses, absorption and scatterings, one obtains even much shorter cavity photon lifetimes and higher gain coefficients. It means that the photon population in the cavity is changing very rapidly. The photon number (power) is kept by the pumping that induces stimulated emission photons which replace in steady state the lost photons.

Broadened mode-spectra resulting from (amplified) spontaneous-emission (is random phase noise) and stimulated-emission, such as those obtained with the transverse-modes in the dye-filled microcavity experiment [3,4], basically exist without thermal excitations, at any temperature. Such spectra will not "cool-down" to a single frequency by lowering the temperature, say to ~0°*K*, if it were not a single-mode lasing but a thermal-BEC effect. (Therefore, it would be crucial to measure in experiments like Ref. 3 the temperature dependence of the spectrum in a broad temperature range, and not only at or close to room temperatures.) There can be thermal effect in the spectra, since the dye molecules [3,4] alone can have within their sub-levels some thermal distribution (with questions and limitations [24]) which affects the photon emission. The microcavity experiment however is not a free dye-molecules system. Therefore the spectral filtering is not only the dye molecules absorption-emission part upon which the Kennard-Stephanov (KS) relation [24] is based. Cavities impose on the photons their own additional rules.

The collapse to a single frequency at the red side of the band in the micro-cavity experiment [3] is supposed to support the photon-BEC view. BEC dictates condensation at the lowest possible frequency (the extreme "red" side. Therefore, if there is further availability in the spectrum at the red side (of the "cut-off"), the BEC peak had to move to there. It is therefore important to confirm that the spectrum cannot extend to even lower frequencies. In the microcavity experiment [3,4], it seems that there can be a potential spectral region of transverse modes below the semi "cut-off". (The short cavity makes the longitudinal modes far apart but doesn't fully cut the spectrum). It is the region of the high transverse modes of the next (to the "red" side) longitudinal mode. Therefore, in a BEC view, the condensation frequency has to move to even lower frequencies. Otherwise, one has to talk about isolated "spectral islands" free of other than thermal constraints, where photons might have regional thermal equilibrium. This possibility would need however further study and verification. In the lasing LC view however it can be simple. The idea there is that the loss-scale, and not the frequency as it is in BEC, determines the light spectrum in a laser cavity. The red side of the cut-off has higher losses (high transverse modes of the next red longitudinal mode) and so the light there is damped. It therefore supports the laser LC approach for the microcavity experiment. The spectrum with the cut-off due to the far apart longitudinal modes provides the needed abrupt loss spectral-filtering profile for the LC phenomenon. Moreover, we have

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already argued that even much less restrictive condition on the filtering leads to laser condensation in two-dimensional laser systems.

LC can thus explain the microcavity experimental results [3], including the spectra below and above the collapse to a single frequency oscillation, associating the low mode population near the frequency semi-cutoff as lasing or LC at the lowest-loss transverse mode, and not as photon BEC. We also stress that LC can produce and explain exponentially dependent spectra, similar to the thermal behavior that was attributed to Bose-Einstein distribution [3,4], as we have shown above (Eq. (5) and Fig. 2b) for common exponential loss (absorption and gain) spectral filtering functions.

We mention again that photon densities in single-frequency lasers can be as high as, and higher, than what was calculated for a photon-BEC realization, where it was said that “photon wave packets spatially overlap” [3]. One may raise questions about that, and argue if there is any new (quantum?) meaning there and in other regular (“classical”?) high photon density systems, such as LC and single-frequency lasers.

We conclude this discussion by noting that although we raised questions on experiments on photon-BEC, we don’t exclude the possibility to observe it. We think however that the whole issue would need further discussion and study, especially of properties that differentiate BEC from single-frequency lasing and LC.

Realization of LC can be obtained in many-mode laser systems. In particular, fiber lasers (one dimensional), such as erbium-doped lasers, are very suitable since they can have many and dense longitudinal modes in their gain bandwidth. For the loss spectrum, it is possible to fabricate synthesized fiber gratings with reflection frequency profiles needed for condensation. So can be used other lasers such as the 2-dimensional microcavity [3], relying on the transverse modes. In this case, LC condensation can result from either spectral filtering or mode filtering and spatial loss modulation [8]. Abrupt spectra can be produced by the frequency semi-cutoffs of far apart longitudinal modes in microcavities, while mode filtering and spatial modulation by lateral confinement. LC is then expected to occur in transverse mode systems [3], as was discussed above.

4. Conclusion

We have presented a classical laser light condensation phenomenon (LC) which is based on a loss hierarchy in a linear mode system with noise and certain spectral filtering. We discussed experimental sides, differences between LC and BEC and difficulties to observe regular photon-BEC in laser cavities. Further experimental study is needed to verify all those issues.

We finally note that the formalism that leads to LC, based on Eq. (1), can have a broad scope. It presents a generic many “particle” Langevin equation with friction coefficients composed of an independent dispersive part ε_m, and a shared part (g). Besides lasers, we can think of various mechanical or biological particles in fluids which follow this model and can therefore show condensation behavior.

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