Laser light condensate: experimental demonstration of light-mode condensation in actively mode locked laser

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Abstract: We have recently predicted (R. Weill, B. Fischer and O. Gat, *Phys. Rev. Lett.* **104**, 173901, 2010) condensation of light in actively mode locked lasers when the laser power increases, or the noise, that takes the role of temperature, decreases. The condensate is characterized by strong light pulses due to the dominance of the lowest eigenmode ("ground state") power. Here, we experimentally demonstrate, for the first time, light mode condensation transition in an actively mode-locked fiber laser. Following the theoretical prediction, the condensation is obtained for modulations that have a power law dependence on time with exponents smaller than 2. The laser light system is strictly one dimensional, a special opportunity in experimental physics. We also discuss experimental schemes for condensation in two- and three-dimensional laser systems.

OCIS codes: (140.4050) Mode-locked lasers; (000.6590) Statistical mechanics.

References and links

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Bose-Einstein condensation (BEC) has been experimentally demonstrated in many systems of cold atoms [1–3]. Condensation was also attributed to classical waves and light such as in weakly nonlinear medium [4], and random media [5]. In the present work we show experimental BEC demonstration in a laser light system.

Our work is done in the realm of a new approach developed for many mode laser systems that is based on statistical mechanics [6–8]. The outcome was a thorough thermodynamics-like theory of statistical light-mode dynamics (SLD), where quantities such as entropy, free energy and noise that takes the role of temperature, are essential for understanding the light system. The theory was applied to passive mode-locking, showing theoretically and experimentally [9–11], that pulse formation is a first order phase transition of the modes from random to ordered phase orientations. The SLD theory was also applied to active mode-locking (AML) [12], the focus of the present work. AML generates pulsation and laser mode ordering by modulating the laser waveform periodically, at a rate that matches or is a multiple of the frequency difference between consecutive axial (longitudinal) modes of the laser [13,14].

Very recently [15], we have theoretically shown the possibility of condensation in AML laser systems (see Fig. 1) when the modulation has a power law dependence on time with an exponent $\eta < 2$. Then Bose-Einstein condensation (BEC) transition was predicted to occur when the power of the laser is increased, by increasing the pumping, or alternatively when the noise in the laser is decreased, by controlling the noise injected into the cavity. In the condensate state the first AML eigenmode carries a macroscopic part of the total cavity power, and the power in all of the higher order eigenmodes, that produces background light, is bounded. The meaning of that will be seen and discussed in the experimental part below.

We summarize here the theory on the optical BEC in AML [15]. The governing equation for AML with noise is given by:

$$\frac{\partial \psi(t,\tau)}{\partial \tau} = \hat{O}(t)\psi(t,\tau) + g\psi(t,\tau) + \Gamma(t,\tau) , \qquad (1)$$

where $\psi(t,\tau)$ is the electric field envelope and t,τ are the short and long time variables describing ψ within a single roundtrip and the propagation dependent in a resolution of the roundtrip, respectively, and $\hat{O}(t) = \left(\gamma_g - i\gamma_d\right)\frac{\partial^2}{\partial t^2} - V(t)$. γ_g and γ_d are the gain filtering and dispersion coefficients, respectively, and g is the saturated net gain. $V(t)$ is the loss-gain modulation, that is a periodic function with a period $t_R = 2L/(c/n)$, the cavity roundtrip time, where L is the cavity length, c - the speed of light and n - the refractive index). The point at which V has its minimal value is denoted by 0. We consider modulations that are not smooth at the minimum with a power law behavior $V(t) \sim M\left|t/t_R\right|^{\eta}$. The commonly used modulation form is sinusoidal that is quadratic in the first order; i.e. $\eta = 2$, characterizing a smooth minimum. Γ, an additive noise term that originates from spontaneous emission and other possible internal and external sources, is modeled by a centered white Gaussian process with covariance $2T$ per unit length.

Equation (1) is solved by an eigenfuction expansion of the operator $\hat{O}(t)$. The main interest is in finding the overall power of the waveform, and compare it with the power of the "ground state" eigenmode; i.e. the lowest pulse mode. We found that in the limit of a large number of bound modes, the overall power is given by:

$$P = \left\langle \int |\psi|^2 \frac{dt}{t_R} \right\rangle = \frac{T}{\varepsilon_0 - g} + T\int_0^M \frac{\rho(\varepsilon)d\varepsilon}{\varepsilon + \varepsilon_0 - g} , \qquad (2)$$

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where the first term at the right hand side gives the power of the lowest pulse eigenmode with an eigenvalue ε_0, and the second term approximates the power in all the higher eigenmodes, with a density of states

$$\rho(\varepsilon) = C_\eta \frac{N}{M}\left(\frac{\varepsilon}{M}\right)^{\frac{1}{\eta}-\frac{1}{2}}, \qquad C_\eta = \frac{1}{4\pi}\int_0^1 \frac{ds}{\sqrt{1-s^\eta}} , \tag{3}$$

where $N = \left(Mt_R^{\,2} / \gamma_g\right)^{1/2}$ is the total number of bound eigenmodes, assumed to be very large. The similarity to BEC can be seen right away [1–5]. According to Eqs. (2) and (3), for small values of P/T the power is distributed among a large number of eigenmodes, and as P increases or T decreases, $\varepsilon_0 - g$ becomes smaller, and the power distribution becomes narrower. For $\eta < 2$ the integral in Eq. (2) converges for $\varepsilon_0 - g = 0$, and a condensation transition takes place for $P > P_c = T\int_0^M \frac{\rho(\varepsilon)d\varepsilon}{\varepsilon}$. When P increases beyond P_c, all the excess power $P - P_c$ resides in the basic eigenmode. However, for $\eta \geq 2$, condensation is not possible and the overall power in all higher eigenmodes that produces a noisy background continue to increase with P/T, deteriorating the coherence properties of the pulse. Theoretical graphs for the lowest pulse eigenmode power $p_0 / P_c(\eta = 1)$ as a function of the total laser power $P / P_c(\eta = 1)$ are given in Fig. 2. We can see the sharp BEC-like transitions for $\eta = 1/2, 1$, and the gradual slow increase for $\eta = 2,\ 4$. An additional statistical-mechanics based insight can be gained when examining the bare axial mode system, perturbed by the modulation that invokes interaction between the modes. For $\eta < 2$ it is a long range mode interaction in the frequency domain, as is the case in passive mode-locking [6–8] and contrary to AML with $\eta \geq 2$ [12], yielding a phase transition even in one-dimensional many body systems. It means in our case mode-phase ordering over the full frequency band and as a result very short pulses.

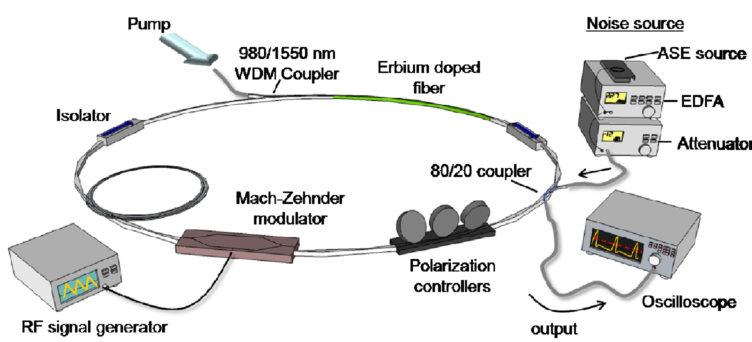

Fig. 1. Actively mode-locked laser: The Experimental setup of the mode-locked ring fiber laser with electro-optic modulator and an external noise source. EDF, erbium-doped fiber; ASE, amplified spontaneous emission.

To experimentally demonstrate the condensation we used a fiber laser. It is advantageous to use in the experiment a long laser cavity with a broad gain bandwidth that supports a large number of modes (*N*). We built a fiber laser with a total length of $\approx 128m$, that corresponds to $641.8nsec$ roundtrip time (a frequency of $1.558\,MHz$). The setup, schematically shown in

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Fig. 1, consisted of an erbium-doped fiber (EDF) ($90cm$ long with $33\,dB/m$ gain) as the gain medium, a Mach-Zehnder modulator, two isolators, and two polarization controllers. The modulator was controlled by RF arbitrary waveform generator, with a resolution of $\sim 0.5\,ns$. Since the shortest pulses width of the lowest pulse eigenmode in the experiment was $\sim 3\,ns$, there was effectively no distortion at the signals edge for $\eta < 2$. Therefore $V(t)$ and $\rho(\varepsilon)$ approximately kept the same power law dependence for small values of x and ε.

In order to have control on the noise strength, an outside noise source was injected into the laser through a 20% coupler. It was taken from the amplified spontaneous emission of an EDF amplifier with 22.7 dBm output, filtered and controlled by a variable attenuator. The measurement of the output light waveform (from the above mentioned 20% coupler) was taken by a photo-diode and a sampling scope. Without the external noise, the laser operated at the pulse mode even for low pumping. For observing the condensation transition, noise with a power of 10.7 dBm was injected into the cavity.

For the active mode-locking we applied modulation signals using the RF waveform generator with power law time dependence with the four different exponents: $\eta = 1/2, 1, 2, 4$. The modulation frequency matched the basic cavity resonance. The laser power was varied for each exponent by controlling the pumping current. Since the power distribution is determined only by the ratio P/T, we could achieve the same effect by varying the noise power ("temperature), as we have formerly done (9,10), instead of the laser power. We recorded and averaged over time the output waveform ($\left\langle |\psi(t)|^2 \right\rangle$) for each pumping level and subtracted from it the biased noise level originating from the unbound modes that are irrelevant in our calculations. We then extracted from it the average total power and the peak power.

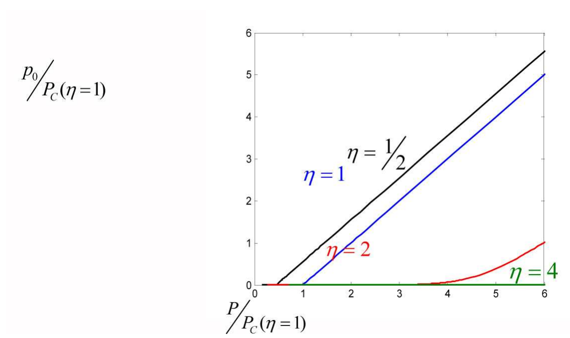

Fig. 2. Theoretical calculation of the AML condensation, following Ref. 4: Graph of the power in the first eigenmode p_0 / P versus the normalized overall laser cavity power $P / P_C(\eta = 1)$, showing sharp transitions for $\eta = 1/2, 1$, and gradual changes for $\eta = 2,\ 4$. The calculation is done for $N = 10^6$.

Figure 3a shows the experimental measurements of the pulse peak power dependence on the total laser cavity power for the four values of η. The peak power gives the pulse squared amplitude that is a measure of the strength of the first eigenmode, and the quality of the mode locking. Figure 3b shows the same graphs for the pulse energy (normalized by $P_C(\eta = 1) \approx 6.7\,pJ$) which is the peak power multiplied by the pulse width. These are the widths of the lowest eigenmode, calculated for each η, that were found to accurately match the experimental results. The graphs in Fig. 3b for the pulse energy show the first eigenmode occupancy. The similarity to the theoretical results in Fig. 2 is evident. The condensation occurs, as the theory predicts [15], for the modulation signals with $\eta = 1/2, 1$, but not for

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$\eta = 2, 4$. The solid lines in Fig. 3 are the theoretical fits to the experimental results, numerically calculated from the waveforms and eigenfunctions expansion for $N \approx 1000$. The experimental results follow very closely the theory. We can also see the agreement with the theoretical prediction for the ratio $P_C(\eta = 1) / P_C(\eta = 1/2) = 6C_1 / 2C_{1/2} = 9/4$, which is close to the measured value of ≈ 2.

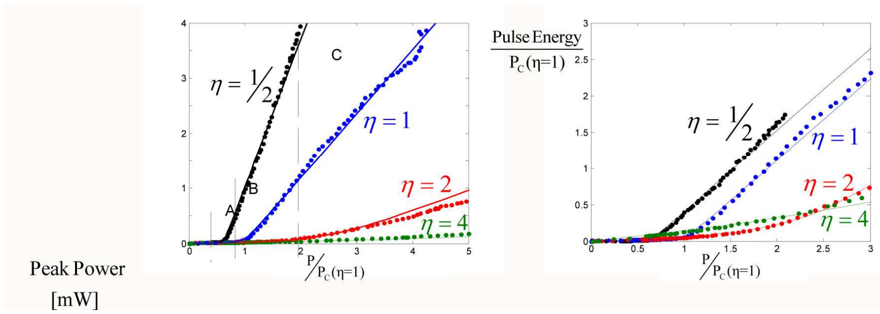

Fig. 3. Experiment showing the condensation: The dependence on the overall laser cavity power normalized by $P_C(\eta = 1) \approx 6.7\ pJ$ of: (a) The measured normalized pulse peak power, and (b) the pulse energy, given by the peak power multiplied by the width of the lowest eigenmode for each η taken from the calculation that was found to match the experimental values in Fig. 4. It measures the first eigenmode occupancy. We can see the similarity to the theoretical graphs in Fig. 2, with a sharp transition for $\eta = 1/2, 1$, and a gradual slow growth for $\eta = 2, 4$. The solid lines are the theoretical fits to the experimental results, numerically calculated from the waveforms and eigenfunctions expansion for $N = 1000$.

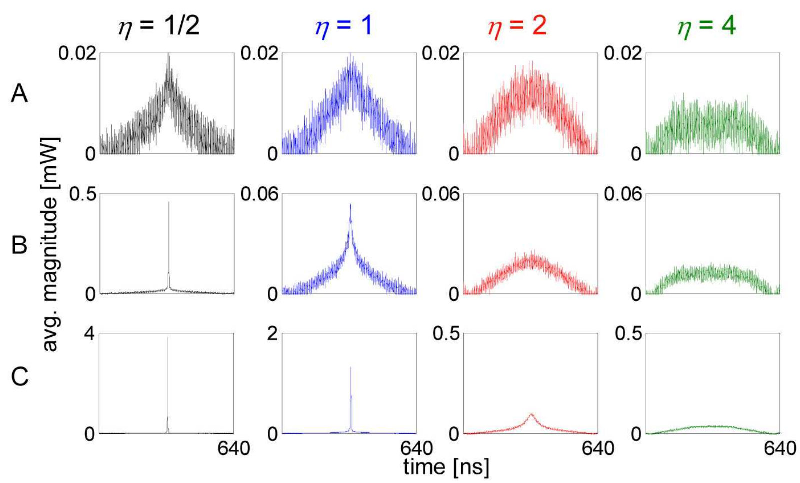

Fig. 4. Measured output light waveforms: Shown for $\eta = 1/2, 1, 2, 4$ at three power levels marked in Fig. 3: (A) $P / P_C(\eta = 1) \approx 0.4$, (B) $P / P_C(\eta = 1) \approx 0.8$, and (C) $P / P_C(\eta = 1) \approx 1.95$. The condensation is seen through the transition from low-amplitude noisy waveforms to high peak power pulses with ~1ns widths for $\eta = 1/2, 1$. The cavity roundtrip time is $t_R = 641.8 nsec$. Note the different power scale in the vertical axis for the various total cavity power.

Figure 4 shows the measured waveforms taken by the oscilloscope, for each of the four values $\eta = 1/2, 1, 2, 4$, at the following three power levels, marked in Fig. 3a: (A)

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$P / P_C(\eta=1) \approx 0.4$, before the transition, (B) $P / P_C(\eta=1) \approx 0.8$, just after the condensation of $\eta = 1/2$, and (C) $P / P_C(\eta=1) \approx 1.95$, when the peak power of $\eta = 2$ starts to increase. In the first row (A), the laser still works in a noisy regime for all η, and the light waveforms show modulated noise that approximately follows the applied modulation signals, each with its respective η, but not mode-locking. The second row (B) shows the condensate waveforms for $\eta = 1/2$, and the non-condensate waveforms for $\eta = 1, 2, 4$ with a gradual population buildup. The third row (C) shows the pulses obtained for all three cases at a high power level, but clearly shows the much better quality of the pulses for $\eta = 1/2, 1$ compared to the $\eta = 2, 4$ cases. We can therefore summarize that when condensation takes over the pulses drastically shorten from noise waveform widths in the order of the cavity length down to ~1 ns)and in shorter lasers and reduced jitter to less than ps), corresponding to the lower eigenmode that depends on η. Since harmonic modulation ($\eta = 2$), the common and simple method used for active mode locking, is at the boundary of condensation, it is a less effective way to generate short pulses compared to the nonsmooth modulations with $\eta < 2$.

We add a note on condensation of light in mode locked lasers at dimensions higher than one. Theoretically, if the $t = x/(c/n)$ dependent operator $\widehat{O}(t)$ in Eq. (1) is extended to 2 or 3 dimensions, i.e. $\widehat{O}(\vec{r}) = (\gamma_g - i\gamma_d)\nabla^2 - V(\vec{r})$, the analysis gives weaker conditions on the potential (modulation) exponent, η, required for condensation. For example, in three dimensions, condensation occurs for all values of η, including the zero potential case where the system has only a volume confinement. For observing condensation in three dimensions we can use for example a laser cavity with many transverse modes instead of a single mode fiber. Under the paraxial approximation, Eq. (1) holds with

$$\widehat{O}(t, y, z) = (\gamma_g - i\gamma_d)\frac{\partial^2}{\partial t^2} + (\gamma_1 + i\gamma_2)\nabla_\perp^2 - V_x(t) - V_\perp(y, z) , \tag{4}$$

where $t = x/(c/n)$ is the propagation direction (in the pulse frame), and y, z are the transverse directions. The $i\gamma_2\nabla_\perp^2$ term naturally originates from the paraxial approximation, whereas $\gamma_1\nabla_\perp^2$ can result from spectral filtering in the transverse spatial frequencies. $V_\perp(y, z)$ is an optional loss potential in the transverse direction, while $V_x(t)$ is the modulation potential. We can think of three-dimensional experimental realization by using a perfectly coated cylindrical cavity, a gain medium with current modulation, and two loss masks: one that introduces a modulated loss $V_\perp(y, z)$ at the transverse directions, and the second mask with matching lenses provide spatial filtering. According to the theory, for any applied modulation signal $V_x(t)$, the laser should produce a condensate waveform, beyond a threshold pumping level, where the first transverse mode and the lowest pulse mode occupy a macroscopic portion of the power.

Conclusion

We have experimentally demonstrated Bose-Einstein condensation in active mode-locking. Besides the basic side in being a new one dimensional laser light BEC system, it can have practical meanings for ways to use modulations that can be more effective than the usual methods for producing high quality short laser light pulses.

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