Gain balance of pulses and noise in passive mode locking with slow saturable absorber

Michael Katz,[1] Baruch Fischer,[1,*] and Omri Gat[2]

[1] *Department of Electrical Engineering, Technion-IIT, Haifa 32000, Israel*
[2] *Racah Institute of Physics, Hebrew University, Jerusalem 91904, Israel*
**Corresponding author: fischer@ee.technion.ac.il*

Received April 7, 2009; revised May 27, 2009; accepted June 22, 2009;
posted June 25, 2009 (Doc. ID 109771); published July 20, 2009

We employ a recently developed gain balance principle to study the problem of passive mode locking with a slow saturable absorber in the presence of noise and solitonic pulse compression. We calculate the compression of the chirped pulse under general conditions and show that there is a minimal achievable pulse width owing to stability requirements. We derive the slow-absorber mode locking parameter, which must exceed a pulse-width-dependent minimal value to sustain mode locking, and calculate the fraction of the total intracavity power that resides in the pulse. We show that choosing the system parameters in an attempt to achieve shorter pulses reduces the pulse power, which, in contrast to fast-absorber passive mode locking, can attain arbitrary small values. Finally, we discuss the modification of the continuum stability condition needed to account for the effect of noise. © 2009 Optical Society of America

OCIS codes: 140.3430, 140.4050.

Generation of ultrashort pulses in passively mode-locked lasers is often based on a *slow* saturable absorber, where the obtained pulse duration is shorter than the absorber's recovery time [1–3]. While the mechanisms of mode locking with a slow absorber have been discussed in several studies [4–8], and a solution of the mode-locking equation in the case of weak absorber saturation is well known [9], the effect of noise on the generation of the mode-locked pulses has not been fully accounted for. Though the noise intensity is much smaller than the intensity of the short pulse, an ordinary perturbation approach is inadequate, since the noise resides in a cavity of length that is much larger than the pulse width, thus rendering the pulse and the noise-continuum powers comparable. This not only alters the steady-state mode-locked pulse power but also raises questions regarding the statistical stability of the mode-locked operation. These questions have been recently addressed via the statistical light-mode dynamics (SLD) approach, which provides a comprehensive description of the pulse-noise interaction when potential conditions hold [10–14]. For general system configurations, the pulse formation in a noisy environment can be studied via the *gain-balance* method [15], which yields results pertaining to the condition of the pulse existence and the calculation of its power. Previously [15], the gain-balance method has been successfully employed in the case of fast saturable absorber mode locking, where it has been connected to the rigorous analysis of SLD.

This method is based on the assumption that the system behavior can be described as a combination of two parts—the short pulse and the noise-generated continuum. We assume that the gain is saturated on a time scale much slower than the round trip, stabilizing the total intracavity power to a constant value $1/L\int_0^L|\psi|^2\mathrm{d}z=P$, where L is the cavity length. This power is divided between the pulse and the continuum, and their dynamics is coupled via the intracavity gain. Since the round-trip gain is uniform, the power distribution is established by imposing the same gain on each of the waveform parts. The validity of the pulse-continuum decomposition is supported on the length scales of our model. Since the pulse occupies only a small fraction of the cavity length, the contribution of the pulse waveform fluctuations to its steady-state power is of order L_p/L (where L_p is the pulse width) and is therefore negligible. Similarly, the nonlinear dynamics of the saturable absorber after the pulse passage, while important for stability calculations, leads to a small correction of order τ/L (where τ is the recovery time of the absorber) to the steady-state noise power distribution.

In the present work, we study the formation of pulses in the regime of weak absorber saturation with dispersion and Kerr nonlinearity, i.e., solitonic effects, which are required to ensure the stability of the continuum in the gain window following the pulse via a high dispersion value [2,5,6].

The pulse-dynamics equation (time evolution is normalized to the cavity round-trip time) is [9]

$$\dot{\psi}=(\gamma_g+i\gamma_d)\frac{\partial^2\psi}{\partial z^2}+i\gamma_k|\psi|^2\psi-(q-q_0)\psi+g\psi, \qquad (1)$$

where γ_g models the gain parabolic spectral filtering, γ_d is the chromatic dispersion, γ_k accounts for the Kerr effect, and g is the overall net gain that includes the amplifier gain, small signal absorption, and other losses. We assume that the absorber recovery time τ is much longer than the pulse width L_p and much shorter than the cavity length L. Then the absorption is $q=q_0\exp(-\int_0^z|\psi|^2/W_s\mathrm{d}z)$, which can be expanded, under the assumption that the pulse energy is much smaller than the saturation energy of the absorber W_s,

$$q = q_0 - \frac{q_0}{W_s}\int_0^z |\psi|^2 \mathrm{d}z + \frac{q_0}{2}\left(\frac{1}{W_s}\int_0^z |\psi|^2 \mathrm{d}z\right)^2. \quad (2)$$

The total intracavity energy is PL, so that the small parameter associated with the weakness of the saturation is $\epsilon = PL/W_s$. The net gain increase due to absorber saturation, or absorber modulation per round trip, is then $g_s = q_0\epsilon$.

The solution of Eq. (1) can be obtained by introducing the chirped-pulse ansatz $A\,\mathrm{sech}^{1+i\beta}(xz/L_p)$, where $A = (\frac{1}{2}x^2PL/L_p)^{1/2}$ and $0 < x < 1$ is the fraction of the total power that resides on the pulse. A steady-state solution of Eq. (1) contains in addition a frequency shift, round-trip pulse displacement, and phase accumulation [7]. Since it can be shown that these terms have negligible influence on the gain balance, we omit them. The chirped-pulse ansatz implies the following equations:

$$(2 - \beta^2)\gamma_g - 3\beta\gamma_d = \frac{\epsilon g_s L_p^2}{8}, \quad (3)$$

$$(2 - \beta^2)\gamma_d + 3\beta\gamma_g = \gamma_k \frac{PLL_p}{2}. \quad (4)$$

We are interested in the case where continuum stability is achieved by strong dispersive effects [2,5]. Then $\gamma_d \gg \gamma_g$ and the chirp is weak, $\beta \ll 1$. We therefore define $\gamma_g = \epsilon\tilde{\gamma}_g$ and $\beta = \epsilon\tilde{\beta}$.

It follows from Eq. (4) that $L_p = 4\gamma_d/(\gamma_k PL)$. The pulse compression can be quantified by comparing L_p with the width of the pulse determined by the saturable absorber without the soliton effects $l_p = 4\sqrt{\tilde{\gamma}_g/g_s}$. Defining the pulse-width compression ratio $\alpha = L_p/l_p$, Eq. (3) gives the chirp, $\beta = \frac{2}{3}\gamma_g/\gamma_d(1-\alpha^2)$.

A zero chirp pulse with the pulse width equal to l_p is therefore obtained when $\alpha = 1$ (i.e., at soliton condition) and a particular case of interest is $\alpha \ll 1$, which means that the dispersion and Kerr coefficients are chosen to obtain significant pulse compression.

The steady-state conditions for the chirped pulse imply that the overall net gain is

$$g = -\epsilon\frac{\tilde{\gamma}_g - 2\tilde{\beta}\gamma_d}{(\alpha l_p)^2}x^2 - \frac{xg_s}{2}. \quad (5)$$

For $\alpha = 1$, as well as in the case of no imaginary coefficients, the first term in the right-hand side is negligible by the assumption $\epsilon \ll 1$ of small absorber saturation, and the net gain $g = -xg_s/2$ is always negative. On the other hand, when $\alpha \sim \sqrt{\epsilon}$ both terms are of the same order of magnitude. Noise stability implies that the net gain should be negative, and the pulse compression is therefore bounded by $\alpha > \alpha_{\min} = \sqrt{\epsilon x/24}$. The expression for the minimal possible pulse width is then

$$L_{\text{p-min}} = \left(\frac{2}{3}\frac{\gamma_g}{g_s}\right)^{1/2}. \quad (6)$$

We now consider the dynamics of the continuum waveform ψ_c away from the pulse. We assume that the absorber saturation reaches its maximal value after the passing of the pulse, and its behavior is given by exponential relaxation with characteristic length τ [5],

$$\dot{\psi}_c = (\gamma_g + i\gamma_d)\frac{\partial^2\psi_c}{\partial z^2} + g\psi_c + xg_s e^{-z/\tau}\psi_c + \Gamma. \quad (7)$$

Γ is a white Gaussian noise with the autocorrelation function $\langle\Gamma^*_{z_1,t_1}\Gamma_{z_2,t_2}\rangle = 2T\delta(z_1 - z_2)\delta(t_1 - t_2)$, where T is the total noise-power injection rate.

Certain conditions must be met by the system parameters in order that ψ_c remains bounded, as discussed below. Assuming that these stability conditions hold, we proceed to calculate the continuum power. Since $\tau \ll L$, the positive gain window opened by the pulse is narrow, and its effect on the noise power is negligible, so we can use results derived with uniform net gain [15]: $g_c = -T^2/(4\gamma_g P^2(1-x)^2)$.

The principle of gain balance asserts that the pulse-power fraction x is determined by equating the noise gain g_c to the pulse gain obtained in Eq. (5). We therefore obtain an equation for the pulse-power fraction:

$$2M(x - rx^2)(1 - x)^2 = 1, \quad (8)$$

where the mode-locking parameter is defined as $M = \gamma_g g_s (P/T)^2$. The parameter $r = \epsilon/(24\alpha^2) = L_{\text{p-min}}^2/L_p^2$ determines the minimal value $M_{\min}$ that is required for the existence of a real solution $0 < x < 1$ of Eq. (8). The exact expression for the mode-locking condition can be approximated for $0 < r < 1$ by $M > M_{\min} \sim 27/8 + 9r/8 + r^2/8$, while for large r it is $M > M_{\min} \sim 2r$.

Equation (8) yields closed-form algebraic expression for the pulse-power fraction x. The dependence of x on the parameters M^{-1} and r is shown in the top part of Fig. 1. Since the solution of Eq. (8) has to satisfy $xr \leq 1$, the maximal pulse power, achievable in the limit of negligible noise, is given by $1/r$ for $r > 1$. Then the decreasing pulse power fraction maintains the pulse width at its minimal value, in accordance to the area theorem.

The minimal pulse-power fraction (achieved for $M = M_{\min}$) at the the soliton condition $\alpha = 1$ is $x = 1/3$ and decreases to zero for increasing values of r, in contrast with the fast saturable-absorber case, where it was found to be half of the total intracavity power independently of the system parameter values [15].

To compare the results obtained under the assumption of the slow absorber saturation to the mode locking of pulses that are much longer than the absorber recovery time, we define the nonlinear absorption coefficient, as for a fast absorber, $\gamma_s = q_0/I_s$, where $I_s = W_s/\tau$ is the saturation intensity, and noise-power injection per mode $\tilde{T} = T/L$. Then, the mode-locking parameter can be expressed as $M = (\gamma_s P^2/\tilde{T})^2(3L_{\text{p-min}}/2\tau)^2$, compared with $M = (\gamma_s P^2/\tilde{T})^2$ in the case of the fast saturable absorber

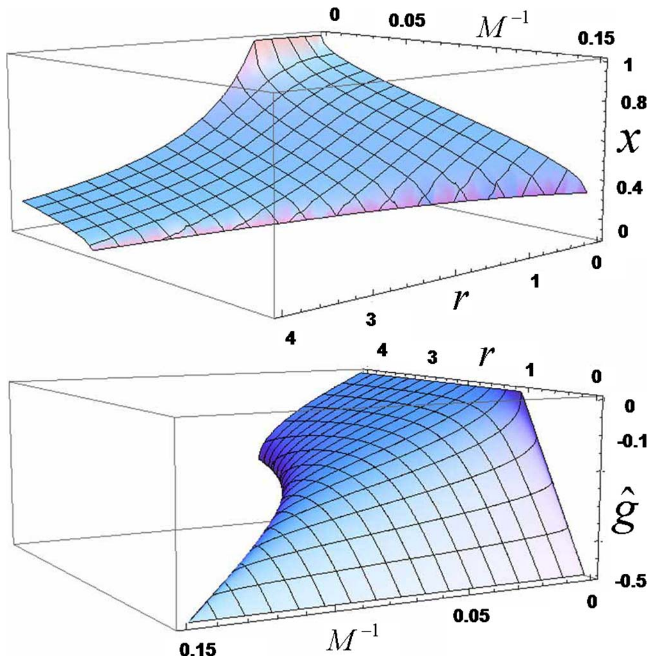

Fig. 1. (Color online) Top, the pulse power fraction, as a function of the inverse of the mode-locking parameter M^{-1}, which defines an effective noise power, and r, the ratio of the minimal pulse width to the pulse width defined by the soliton effects. The pulse ceases to exist when $M<M_{\min}$. Note the maximal power limit for $r>1$. Bottom, the normalized net-gain value that determines the continuum stability condition.

[15]. Since the pulse width is much shorter than the absorber response time, the pulse sustainability condition is much more stringent for a slow saturable absorber mode locking.

Finally we address the issue of continuum stability that is determined by the homogeneous part of Eq. (7). For the continuum modes to be stable, the eigenvalues of the linear operator on the right-hand side of Eq. (7) should have a negative real part. In quantum-mechanics language, this condition states that the energy of the bound states of the exponential potential well of depth xg_s, created by the pulse, should be above the (negative) value of the net gain g. It has been analyzed thoroughly in previous studies [2,5] where the necessary and sufficient condition for stability for a given g was derived. However, these studies have not accounted for the role of noise in determining the net-gain value g.

In our model the net gain is derived from the gain balance, and it can be expressed via the pulse-power fraction x that sustains Eq. (8): $g=-g_s/(4M(1-x)^2)$. The net gain relative to the potential well depth is given by $\hat{g}=g/(xg_s)$, and its dependence on M^{-1} and r is shown in the bottom part of Fig. 1. Since the bound-mode energy value depends on γ_d, the continuum stability demand results in a condition on the minimal value of the dispersion parameter. We note that in the case of no soliton effects, the net-gain value is always $\hat{g}=-1/2$; however, the bound-mode energy is determined by γ_g alone, and it becomes impossible to concomitantly achieve continuum stability and mode-locked operation.

In conclusion, we have shown that to achieve a minimal pulse width without imposing a constraint on pulse power, the dispersion to Kerr nonlinearity ratio has to be equal to an optimal value, according to Eq. (6). We have obtained that the time-bandwidth product of the pulse duration with the gain spectral width is inversely proportional to the square root of the absorber modulation per round trip. Since our model assumes weak absorber saturation, taking a large-modulation-depth saturable absorber (for example, $g_s\sim 0.04$) limits the pulse to about one-fourth of the available spectral width. We have also obtained the relation for the mode-locking parameter, which determines the minimal ratio of the intracavity power to the noise power within the laser bandwidth, necessary to sustain mode locking. The required ratio is inversely proportional to the square root of the absorber modulation per round trip as well, and, for the above-mentioned optimal configuration, amounts to approximately 20. As the lower noise bound of spontaneous emission is tens of microwatts in a typical fiber laser, the mode-locking threshold power can be estimated as 1 mW. This power is comparable with the values required in the fast absorber mode locking, since the effective nonlinear coefficient of a slow absorber can be tens of times stronger than in the in-fiber implementations of a fast absorber, such as the one obtained by nonlinear polarization rotation. However, if the same absorber were employed in the two mode-locking cases: one where the pulses are shorter than the saturation recovery time and the second where the pulses are much longer than that time (then the absorber response can be regarded as instantaneous), the first case would require much higher intracavity threshold power.

References

1. I. D. Jung, F. X. Kartner, L. R. Brovelli, M. Kamp, and U. Keller, Opt. Lett. **20**, 1892 (1995).
2. F. X. Kartner, I. D. Jung, and U. Keller, IEEE J. Sel. Top. Quantum Electron. **2**, 540 (1996).
3. L. W. Shi, Y. H. Chen, B. Xu, Z. C. Wang, Y. H. Jiao, and Z. G. Wang, J. Phys. D **40**, R307 (2007).
4. E. P. Ippen, Appl. Phys. B **58**, 159 (1994).
5. F. X. Kartner and U. Keller, Opt. Lett. **20**, 16 (1995).
6. F. X. Kartner, J. A. der Au, and U. Keller, IEEE J. Sel. Top. Quantum Electron. **4**, 159 (1998).
7. N. N. Akhmediev, A. Ankiewicz, M. J. Lederer, and B. Luther-Davies, Opt. Lett. **23**, 280 (1998).
8. R. Paschotta and U. Keller, Appl. Phys. B **73**, 653 (2001).
9. H. A. Haus, IEEE J. Sel. Top. Quantum Electron. **6**, 1173 (2000).
10. A. Gordon and B. Fischer, Phys. Rev. Lett. **89**, 103901 (2002).
11. A. Gordon, B. Vodonos, V. Smulakovski, and B. Fischer, Opt. Express **11**, 3418 (2003).
12. O. Gat, A. Gordon, and B. Fischer, Phys. Rev. E **70**, 046108 (2004).
13. R. Weill, A. Rosen, A. Gordon, O. Gat, and B. Fischer, Phys. Rev. Lett. **97**, 013903 (2005).
14. A. Gordon, O. Gat, B. Fischer, and F. X. Kartner, Opt. Express **14**, 11142 (2006).
15. M. Katz, O. Gat, A. Gordon, and B. Fischer, Phys. Rev. Lett. **97**, 113902 (2006).