Localization of light in a random-grating array in a single-mode fiber

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We investigate light propagation in randomly spaced fiber gratings in a single-mode fiber and demonstrate the localization effect. Localization of light in random media resembles that of electrons in disordered solids, resulting from a subtle wave-interference formation. We measured the light transmission after each additional grating fabrication and found an exponential decay that follows the localization theory. Important features of the random array are its similarity to ordered gratings in the transmission and its reflection behavior at the long-array regime. Besides the basic interest in localization in one-dimensional systems, random-grating arrays have potential applications, utilizing the possibility of the fabrication of long structures with strong and broadband reflections. © 2005 Optical Society of America

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1. INTRODUCTION

Wave propagation in random media has been an important research topic throughout the years, gaining much attention when the concept of localization appeared and stimulated a large amount of work. The idea of localization was first raised by Anderson[1] for electrons in disordered solids that were drastically affected by quantum mechanical wave interference.[2,3] The quest to study and realize such effects in optics was natural, and indeed a considerable amount of research can be found on light propagation in random media that include aspects of localization. Using light, in lieu of electrons, for the study of localization adds new possibilities, mainly in the experimental aspects, owing to the relatively easy measuring techniques and the direct access to the optical "wave function" via light-intensity measurement.

Properties of wave propagation in random media, including localization, generally depend on the system dimensionality. The theoretical analysis of a one-dimensional (1D) system is obviously easier than it is for higher dimensions, but is not at all trivial for experimental realizations in solid-state physics. In optics, however, the experimental situation is very different, and 1D wave propagation is simple. There have been many papers on localization aspects with electromagnetic waves in the optical[4–9] and the microwave[10,11] regimes. We point out the work by Berry and Klein[4], which is very relevant to our present study. These authors showed in a simple but remarkable experiment that for a stack of N transparent plates with randomly varying thicknesses the transmitted intensity decays exponentially with N. The striking feature of the random optical elements is that their overall transmissivity τ_N is given by a simple multiplication of the single-element transmissivity τ, i.e., $\tau_N = \tau^N$. This means that only the direct transmission counts, whereas any multiple reflections added in the direction of the transmitted light interfere destructively. We also mention our work on two experimental realizations for localization of light in optical kicked rotors, which resemble the quantum kicked rotor that relates to Anderson localization. In the first case[12,13] we demonstrated localization in the spatial frequency domain of free-space light beam propagation through an array of thin sinusoidal phase gratings. In the second case[14–16] we studied the spectrum (or sidebands) localization of light pulses that are repeatedly "kicked" by a sinusoidal rf modulation along a fiber.

In this paper we present an experimental study of 1D localization by means of light propagation in a random Bragg-grating array fabricated into a single-mode fiber. We demonstrate the localization behavior, manifested in the strong exponential decay of the light transmission along the fiber, that was measured directly after the fabrication of each additional grating. This decay, which results in high reflection, should not be confused with the much-smaller fiber loss. A report on this finding was given earlier.[17] The theoretical analysis of such system is based on the transfer-matrix formalism in which the system is represented by a product of random matrices. The asymptotic behavior of such a product results from a theorem on products of random matrices by Furstenberg.[18] This theorem ensures that under very general conditions, the elements of the matrix product and any norm of the matrix product grow exponentially with the same exponent, giving rise to the localization behavior. We refer the reader to a comprehensive analysis for a 1D disordered system given by Pendry.[3] Besides the basic propagation effects in the random array, the study can have important ramifications on fiber-optic communication and gratings

Fig. 1. Random fiber-grating array.

technology. Examples are strong and broadband reflectors and fiber and random lasers.

The scattering elements in our system are the random gratings in single-mode fibers. Fibers are an ideal experimental medium for 1D light propagation with a matured technology of in-fiber grating fabrication. The gratings were made to be almost identical, with the same and relatively large reflection-wavelength bandwidth of a few nanometers, obtained by fabricating short gratings. Therefore the interesting effects concerning the localization occur within that bandwidth. Gratings are very effective scattering elements such that we could observe the localization effect with a relatively small number of them, about 50 gratings. The randomness of the scattering elements enters by the random spacing between the gratings (see Fig. 1).

The outline of the paper chapters is as follows: We first give in Section 2 a theoretical treatment of the wave propagation in randomly spaced gratings. We then compare the wave-theory result for the light transmissivity to the calculation obtained from the ray theory and also to the wave propagation in an ordered fiber grating. Section 3 describes the experiment, starting with the setup and the grating-fabrication system and then presents the experimental results. We show transmission measurements, the spectra, and the transmissivity as a function of the gratings number for the random fiber system. These curves are the central results of the paper, showing the localization effect in the random-grating array. We then compare the measured results with the theory and find a very good agreement. We end the paper with conclusions and remarks on the application sides.

2. THEORETICAL ANALYSIS OF LIGHT TRANSMISSIVITY IN A 1D RANDOM-GRATING ARRAY

We present a theoretical treatment for wave propagation in a single-mode fiber with N randomly-spaced Bragg gratings and calculate the transmission in the limit of $N \gg 1$ to obtain the localization length. This result readily reveals that the interference among all reflected waves is destructive for the transmission, and an intuitive explanation is presented. We then compare this analysis with ray theory, in which light is treated as lacking phase property, to show that contrary to the exact wave calculation, this theory results in transmission that decays as $1/N$. Finally, we calculate the transmission of an ordered system and compare the decay rates in both cases.

A. Disordered Grating Array
A transmission calculation through a disordered chain of gratings can be carried out by transfer matrices

methods.[11] The basic idea underlying such a calculation assumes that the system can be cut into slices, whereupon each can be easily evaluated. Then, by writing the transfer matrix of the complete system as a product of those matrices, we can apply the Furstenberg theorem[18] to obtain the asymptotic behavior of the product.

The grating system is described in Fig. 1. Light with wave number k propagates along a single-mode fiber having an array of successive randomly spaced gratings. It can be assumed that the space widths d_i are drawn independently from the density distribution function $d\mu(d_i)$, and that the gratings are identical, i.e., that they have the same lengths and refractive indices.

We describe the propagation of the light in the 1D medium by the transfer matrix M_i that relates the amplitudes of the forward- and backward-propagating waves on the right side of each optical element to those on the left side (see Fig. 2):

$$\begin{bmatrix} a_n^+ \\ a_n^- \end{bmatrix} = M_n \begin{bmatrix} a_{n-1}^+ \\ a_{n-1}^- \end{bmatrix}. \tag{1}$$

For optical lossless elements that are invariant under time reversal, the scattering matrix (which relates the amplitudes of the ingoing waves to those of the outgoing waves via reflections and transmissions) is unitary. Then, by denoting the amplitude reflection and transmission coefficients from both sides, for instance from the left r_n, t_n and from the right r'_n and t'_n, we have the relations $r'_n t_n^* + r_n^* t'^*_n = 0$ and $|r_n|^2 + |t_n|^2 = |r'_n|^2 + |t'_n|^2 = 1$, (the asterisk stands for complex conjugate), and the transfer matrix is given by[4,19]

$$m_n = \begin{pmatrix} \dfrac{1}{t_n^*} & -\dfrac{r_n^*}{t_n^*} \\ -\dfrac{r_i}{t_n} & \dfrac{1}{t_n} \end{pmatrix}. \tag{2}$$

The transfer matrices are unimodular (det $\mathbf{m_n}=1$). In our system we define each element as being comprised of one grating and its successive space. The gratings are taken to be identical, and the spacing between them is responsible for the random part. The transfer matrix of such an element is the product of the grating transfer matrix and the space transfer matrix. From a coupled-wave equation of the counterpropagating waves, the transfer matrix for a single grating is given by[20]
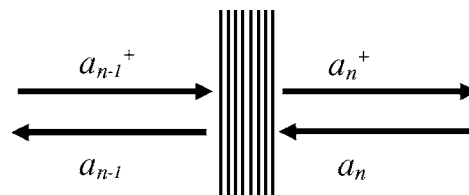

Fig. 2. Incident and reflected field amplitudes that define the transfer matrix of a single grating.

$$m_g = \begin{pmatrix} \cosh(SL_0) - i\dfrac{\Delta\beta}{S}\sinh(SL_0) & -i\dfrac{\kappa}{S}\sinh(SL_0) \\[2ex] i\dfrac{\kappa}{S}\sinh(SL_0) & \cosh(SL) + i\dfrac{\Delta\beta}{S}\sinh(SL_0) \end{pmatrix},$$ (3)

where L_0 is the single-grating length, κ is the coupling coefficient between the counterpropagating beams in the gratings, $\Delta\beta = \beta - \pi/\Lambda$ is the wave-number deviation from the Bragg wavelength, Λ is the grating period, and $S = (|\kappa|^2 - \Delta\beta^2)^{1/2}$. The transfer matrix for a space of length d_i is given by

$$m_{d_i} = \begin{bmatrix} \exp(ikd_i) & 0 \\ 0 & \exp(-ikd_i) \end{bmatrix}.$$ (4)

Then the transfer matrix for a single element is the product of the two above matrices,

$$m_i = m_g m_{d_i} = \left\{ \begin{bmatrix} \cosh(SL_0) - i\dfrac{\Delta\beta}{S}\sinh(SL_0) \end{bmatrix}\exp(ikd_i) \quad -i\dfrac{\kappa}{S}\sinh(SL_0)\exp(-ikd_i) \\[3ex] i\dfrac{\kappa}{S}\sinh(SL_0)\exp(ikd_i) \qquad \begin{bmatrix} \cosh(S\text{Ł}_0) + i\dfrac{\Delta\beta}{S}\sinh(SL_0) \end{bmatrix}\exp(-ikd_i) \right\},$$ (5)

and the single-element transmission and reflection coefficients are given by

$$t_i = [m_i]_{22} = \begin{bmatrix} \cosh(SL_0) - i\dfrac{\Delta\beta}{S}\sinh(SL_0) \end{bmatrix}^{-1}\exp(ikd_i),$$ (6)

$$r_i = -[m_i]_{11}/[m_i]_{22} = \kappa\exp(ikd_i)\sinh(SL_0)/[iS\cosh(SL_0) - \Delta\beta\sinh(SL_0)].$$ (7)

$\alpha_i = kd_i$ provides the random nature of the system when we have a set of such elements. It is assumed that the space widths d_i, are drawn independently from a density distribution function $d\mu(\alpha_i)$. The transfer matrix for N gratings and N spaces is:

$$M_N = m_1 m_2 \ldots m_N = \begin{pmatrix} \dfrac{1}{T_N^*} & -\dfrac{R_N^*}{T_N^*} \\[2ex] -\dfrac{R_N}{T_N} & \dfrac{1}{T_N} \end{pmatrix}.$$ (8)

T_N and R_N are the amplitude transmission and the reflection coefficients, respectively, for the entire system. All coefficients as well as the transfer matrix of the complete optical array are denoted in this paper by capital letters: T,R,M, compared with t,r,m, for one element. For the use below we also denote the intensity transmissivity and reflectivity for a single grating by $\tau = |t|^2$, $t_{,,,} = \tau\exp(\theta)$, and $\rho = |r|^2$; and for the array of N gratings: $\tau_N = |T_N|^2$ and $\rho_N = |R_N|^2$.

We next evaluate the product of those N random unimodular matrices of Eq. (8) to obtain the overall system transmission. The asymptotic behavior of $\mathbf{M_N}$ can be obtained using the Furstenberg theorem[18] on the product of random matrices, stating that under very general conditions, the elements of the matrix product and any normal of the matrix product grow exponentially with the same exponent:

$$\frac{1}{N}\log|m_N \ldots m_1 u| \rightarrow \log\left|\frac{m(\alpha)u}{u}\right| d\mu(\alpha)dv(\theta) \equiv \gamma,$$ (9)

where

$$v[\theta] = \int v[\theta(\alpha)]\frac{d\theta(\alpha)}{d\theta}d\mu(\alpha)$$ (10)

defines the probability distribution of u, and $\theta = \arg(u)$. For the transfer matrix given in Eq. (2), and in fact for more a general case, Furstenberg's conditions are satisfied, as shown by Matsuda and Ishii.[21] Then from Refs. (9) and (10) the exponent is given by

$$\lim_{N\to\infty}\frac{1}{N}\ln\tau_N = -\ln(1/\tau),$$ (11)

where τ is the transmission of a single grating and the system overall transmission is

$$\tau_N = \exp[-N\ln(1/\tau)] = \tau^N.$$ (12)

This simple result reveals an interesting property of the transmission through a set of randomly spaced scatterers in a 1D system by showing that it includes only from the multiplication of the single gratings transmission without all multiple reflections. Of course, reflections were taken into consideration in the above calculation, but asymptotically the result teaches us that all multiple reflections are canceled.
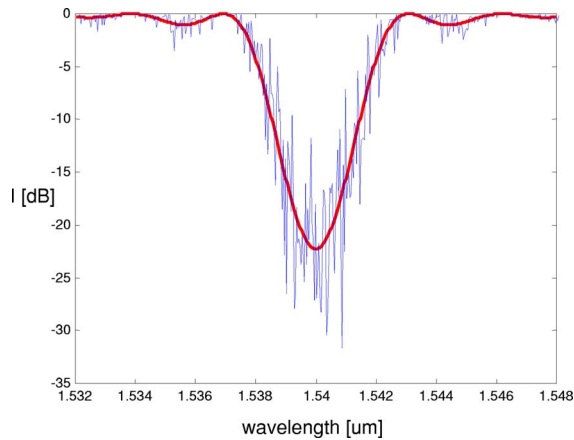
Fig. 3. (Color online) Transmission-spectrum simulation of 1000 randomly spaced gratings with a single-grating transmissivity of 0.022 dB at the band center. The continuous curve shows the asymptotic behavior of the transmission spectrum.
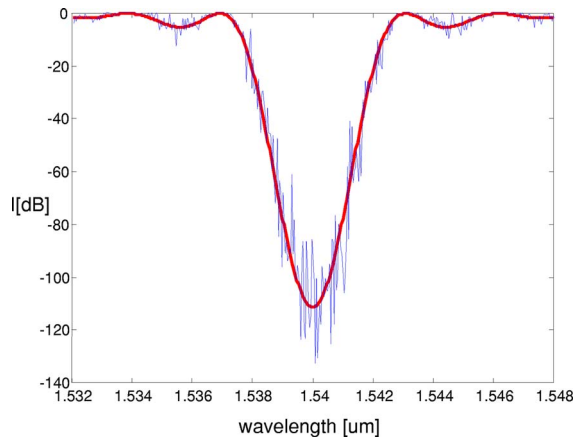

Fig. 4. (Color online) Transmission-spectrum simulation of 5000 randomly spaced gratings with a single-grating transmissivity of 0.022 dB at the band center. The continuous curve shows the asymptotic behavior of the transmission spectrum.
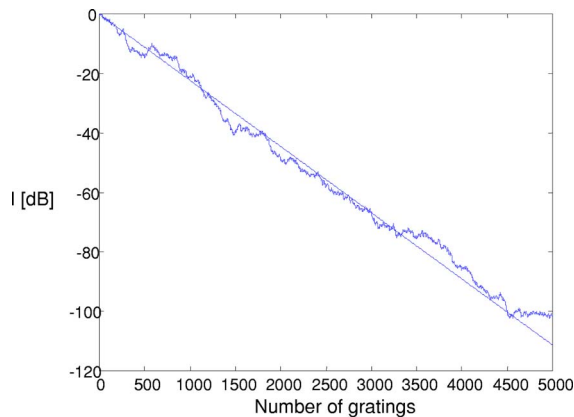

Fig. 5. (Color online) Transmissivity at the band center after each grating. Simulation and the asymptotic behavior (the straight line) given by the theory for a single-grating transmissivity of 0.022 dB.

B. Numerical Simulation for a Disordered Grating Array

In the previous subsection we obtained the transmissivity for a large number of disordered gratings and found it to decay exponentially with the number of gratings. Here we compare this analytical result with a numerical simulation of the transmission. Figures 3 and 4 depict the transmitted intensity spectrum after 1000 and 5000 gratings, respectively. The gratings were taken to be identical and have the following properties: centered at 1540 nm and have a coupling coefficient $\kappa = 185$ m^{-1}, and a length $L = 0.385$ mm. The transmissivity for a single grating at the band center is 0.022 dB. The distances between two successive gratings were chosen randomly from the interval $[0-1]$ mm. The figures also present the transmission spectrum given by Eq. (12), with the wavelength dependent transmission τ of a single grating having the same parameters as those given above. In both cases, a very good agreement was obtained between the analytical calculation and the numerical simulation. The smoother nature of the longer array is obvious, as the averaging action over many gratings is more uniformly spread. One can view the array output spectrum as being composed of all grating pairs making many random Fabry–Perot etalons. The output is the collective spectra, which gradually lose their individual Fabry–Perot characteristic as the light passes more gratings. Figure 5 shows the evolution of the transmitted intensity at the band center after each grating. Here, too, the good agreement between the analytical calculation and the numerical simulation is well observed.

C. Destructive Interference of the High-Order Reflections

It is possible to consider the total transmission as an infinite sum of waves formed by multiple reflections and transmissions consisting of different optical paths and different phases. Figure 6 is an example of a system built out of six randomly spaced gratings. The figure exemplifies three waves with exactly the same overall path length but with a different number of transmissions and reflections. Nevertheless, owing to the phase difference between the reflection coefficient of the forward- and backward-propagating waves, the two upper waves interfere constructively; however, the third wave interferes destructively with the upper two. The fascinating result of
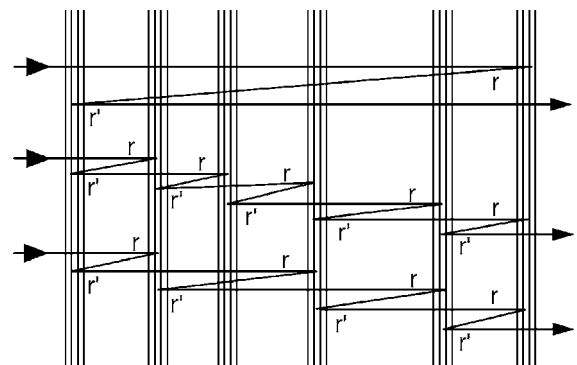

Fig. 6. Three waves with paths of the same length but different numbers of reflections, resulting in constructive interference between the two upper waves but destructive interference with the third.

the localization theory is that for large arrays the overall interference of the all multireflections in the transmitted light is fully destructive. Therefore the total transmission comprises only the wave that passes through all elements without being reflected.

The phase difference between the reflection coefficients for opposite wave incidence at an optical element is general. We are familiar with the opposite sign of the reflection coefficients for opposite incident waves at a boundary of two media with different refractive indices. More generally, the phase difference can be tracked in the relation mentioned in Subsection 2.A that $r'_n t_n^* + r_n^* t'^*_n = 0$. For these optical elements $|r_n| = |r'_n|$, $t_n = t'_n$ and for specific choice of the reference planes of the waves at the two sides of the element, $\arg(t_n) = \arg(t'_n) = 0$, and then we obtain $r_n = -r'_n$.

D. Ray Theory for a Disordered System

We show here the simple ray theory approach that could have been expected to be adequate for the disordered system but in fact leads to wrong results. The development follows the work by Berry and Klein[4] given here to clarify the basic wave nature responsible for the localization effect. Ray-theory approach is based on the assumption that the waves in a disordered system are incoherent and therefore can be represented as intensities rather than amplitudes. The appropriate matrix formalism can be obtained for the ray theory, referring to incident and reflected intensities. When ρ and τ are the one-element intensity reflectivity and transmissivity, where for lossless scatterers $\tau + \rho = 1$, the one-element transfer matrix is

$$m = \begin{pmatrix} \tau - \dfrac{\rho^2}{\tau} & \dfrac{\rho}{\tau} \\[2mm] -\dfrac{\rho}{\tau} & \dfrac{1}{\tau} \end{pmatrix}, \tag{13}$$

and for N successive elements

$$m^N = \begin{pmatrix} \tau - \dfrac{\rho^2}{\tau} & \dfrac{\rho}{\tau} \\[2mm] -\dfrac{\rho}{\tau} & \dfrac{1}{\tau} \end{pmatrix}^N = \left[I + \dfrac{\rho}{\tau}\begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix} \right]^N = I + N\dfrac{\rho}{\tau}\begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix}. \tag{14}$$

I is the unit matrix, and the last equality is based on $\begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix}^N = 0$ for $N \geq 2$. Therefore the ray-theory transmissivity for N random gratings is

$$\tau_N = m_{22}^{-1} = \dfrac{\tau}{\tau + N(1 - \tau)}. \tag{15}$$

It is a linear decay, or Ohmic like behavior for $(1/T_N) \propto N$ (for large N), which is fundamentally different from the exponential dependence results from localization theory. Figure 7 graphically shows the transmissivity in the two approaches. The different result of the ray theory shows the distinction in regarding waves as incoherent and averaging over random phases. This difference is exemplified in Section 2 C; although propagating in a random medium, different light-wave paths of the same
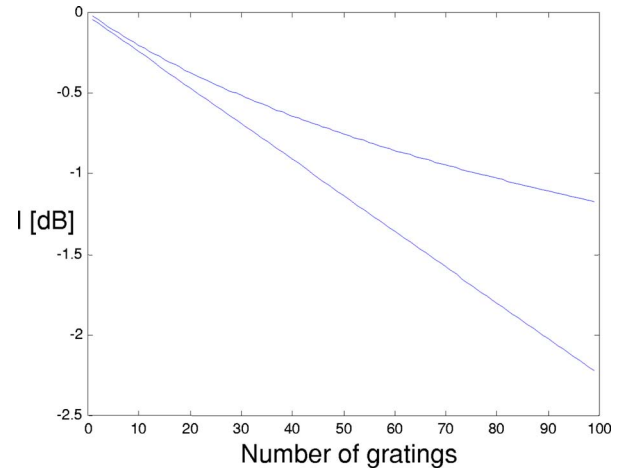

Fig. 7.   (Color online) Comparison between wave and ray theories for the transmissivity in random gratings; the upper curve depicts the Ohmic behavior of the ray theory, whereas the lower curve depicts the exact wave averaging (both calculated for grating transmissivity of 0.022 dB).

lengths "magically" give precise destructive interferences. Thus the assumption that the scattered waves have no phase correlation, and therefore that they can be regarded as incoherent, is false. Furthermore, exact wave averaging shows that all transmitted waves (except for the one passing without any reflection) interfere destructively, leading to the exponential decay of the transmissivity. It is also noteworthy that the ray theory does give a correct result when the wave interference is not dominant. This can occur when the reflection is very small and the system is small enough, so even small reflections would not accumulate.

E. Comparison with Ordered Gratings

It is interesting to compare the random-grating system with ordered gratings. We have a powerful method for obtaining effectively long gratings, with easy creation of random structures that might have been regarded as a detriment, but this turns out to be an advantage with the support of a localization effect that provides a strong transmission decay and high reflection. Then, not only are the random-grating arrays easy to create, but disturbances do not have much of a deteriorating effect on their performance. On the other hand, long ordered gratings are hard to create and can have a detrimental environmental effect. The situation is even worse for ordered-grating arrays, which are almost impossible to implement, even for a small number of gratings, since they need precise interferometric spacing between the gratings. An additional advantage of the random arrays is that they can easily provide very large bandwidths for the reflection, since they depend on the single-grating bandwidth. The single grating can be made very short (10–100 μm long) and still be an effective scatterer, thus providing very large wavelength bandwidths of tens of nanometers. One can then argue that for even larger bandwidths, it is possible to form point scatterers rather then short gratings. Such random point-scatterer arrays are, of course, interesting and worthy of implementation, though they are not easy to make. The array needs the fabrica-

tion of many random scatterers along the fiber. To reach the asymptotic noiseless localization regime, we need a reasonable number of scatterers. This means that the strength of one scatterer ought to be weak to allow the light to penetrate and acquire many multireflections, which averages to zero in the transmitted light. On the other hand, the fabrication of many scatterers is more complicated, and therefore we need a reasonable scattering strength for each element in order for us to be able to observe the effect. There are also some disadvantages of the random array. The transmission and the reflection and their spectra are not smooth and uniform, as the averaging in a random structure (which is limited in length) is not optimal. We need a rather long array to reach the smoother asymptotic behavior. The last point in the ordered–disordered gratings comparison is that gratings are used mostly for precise filtering purposes and not only for reflection, and thus the ordered element is needed, unless we look for special filtering uses, or with special fingerprints.

For a comparison between the disordered array and the ordered structures, like long uniform gratings, we assume them all to be of the same overall length $L = NL_0$. We can also extend the comparison to ordered arrays of N gratings, each of length L_0, with exactly the same spacing between them. We note that the latter structure is almost impossible to implement, even for low number of gratings, because of the subwavelength-spacing requirement.

We use Eq. (3) for the transmissivity of long uniform grating, replacing L_0 with L, and then compare the outcome to the random-grating result. For the ordered-grating array we can again start with the transfer-matrix formalism, using Eqs. (5) and (8) and setting equal spacing, $d_i = d$. This structure includes the ordered-grating case for $d = 0$. Then the transfer matrix for a set of N gratings is

$$M_N = m^N. \tag{16}$$

M_N is given for a unimodular matrix by

$$M_N = \begin{bmatrix} m_{11}U_{N-1}(a) - U_{N-2}(a) & m_{12}U_{N-1}(a) \\ m_{21}U_{N-1}(a) & m_{22}U_{N-1}(a) - U_{N-2}(a) \end{bmatrix}, \tag{17}$$

where U_N are the Chebyshev polynomials of the second type,

$$U_N(a) = \frac{\sin[(N+1)\cos^{-1}a]}{\sqrt{1-a^2}}, \tag{18}$$

and
$a = \frac{1}{2}(m_{11} + m_{22}) = \cosh SL \cos kd + \Delta\beta/S \sinh SL \sin kd$.
Then

$$M_N$$

$$= \frac{1}{\sqrt{1-a^2}} \left\{ \begin{bmatrix} \cosh(SL) - i\frac{\Delta\beta}{S}\sinh SL \end{bmatrix} \exp(ikd)\sin(N\cos^{-1}a) - \sin[(N-1)\cos^{-1}a] - i\frac{\kappa}{S}\sinh(SL)\exp(ikd)\sin(N\cos^{-1}a) \\ i\frac{\kappa}{S}\sinh(SL)\exp(ikd)\sin(N\cos^{-1}a) \begin{bmatrix} \cosh(SL) + i\frac{\Delta\beta}{S}\sinh SL \end{bmatrix} \exp(ikd)\sin(N\cos^{-1}a) - \sin[(N-1)\cos^{-1}a] \end{bmatrix} \right\}. \tag{19}$$

Therefore the transmission coefficient is

$$T_N = [M_N]_{22}^{-1} = \frac{(1-a^2)^{1/2}}{\left(\cosh SL + i\frac{\Delta\beta}{S}\sinh SL\right)\exp(ikd)\sin[N\cos^{-1}(a)] - \sin[(N-1)\cos^{-1}(a)]}. \tag{20}$$

For large N, in all cases (the disordered array, the ordered array with optimal spacing, and the single grating), the long grating regime ($N \gg 1$) for the transmitted intensity at the central wavelength ($\Delta\beta = 0$) is given by

$$\tau_N \propto \exp(-NL_0 S) = \tau^N. \tag{21}$$

The exponent in the ordered case depends strongly on the grating spacing selection. The strongest reflection is given by the single long grating, which has a built-in equal-spacing zero phase; regardless, such arrays are difficult to implement. The great surprise is the result for disordered arrays that gives exponential dependence and accordingly

high reflectivity, although it has its own drawbacks, as we describe in Section 3.

3. EXPERIMENT: SYSTEM, MEASUREMENTS, AND RESULTS

We first describe the setup for the grating fabrication and then the system for measuring the transmission of the randomly spaced gratings. The first setup includes a UV laser that is used for grating fabrication, a set of lenses that are used to shape the laser beam, a mask to create the grating pattern, a moving table and controller for fi-

ber placement, and a single-mode fiber. The second setup is comprised of an erbium-doped fiber amplifier (EDFA) that is used as a source for the transmission measurement and an optical spectrum analyzer for conducting the measurements. We then present the measured results that include the transmission spectral behavior and the dependence on the number of gratings. We also show measurements for deducing the one grating transmissivity, needed for the theory verification. At the end of this section we discuss the results with a comparison with the localization theory.

A. Grating Fabrication and Measurement Setup

The method used to fabricate the gratings is based on a near-contact exposure through a phase mask.[22] The setup is illustrated in Fig. 8. The UV laser source is an argon–ion laser whose frequency is doubled by a nonlinear crystal to give a wavelength of 248 nm and power of approximately 200 mW. The beam is broadened by a concave lens in order to produce a spot size large enough to illuminate the slit on the mask as uniformly as possible without significantly reducing the beam intensity. Then the beam is focused in the fiber axis by a cylindrical lens in order to maximize the intensity, exposing the fiber. The beam emerging from the cylindrical lens is normally incident on a slit attached to the mask and transfers only 1 mm of the beam (the slit is adjusted to pass the interval with the maximum intensity). The slit and the mask are placed approximately at the focal point of the cylindrical lens to achieve the maximum intensity on the fiber that is adjacent to the mask. The exposing beam is then normally incident on the phase mask and diffracted entirely. The grating is formed by the interference between the +1 and −1 diffracted orders of the phase mask. A single-mode fi-

ber was put inside a high-pressure hydrogen tank for a few days to make it sensitive to a photoinduced refractive index change.

B. Light Transmission Measurements

For reproducibility of the grating spectrum, it was necessary to ensure that the exposure time would be similar for all gratings. Therefore the laser power was adjusted to achieve a relatively slow grating formation time in order to render good accuracy.

The following procedure was repeated for each grating:
- The illumination was started.
- After 1 min, the illumination was stopped and the spectrum was recorded.
- The automated stage controller was adjusted to move the fiber holding stage a random distance that was larger than the grating.
- The above procedure was repeated 55 times.

It is noteworthy to mention that the maximum number of 55 gratings was due to limitations of the spectrum-analyzer accuracy, as the transmitted light intensity decreased with the grating number. The measurement results of the spectrum as well as the intensity are shown in Figs. 9–12. The experiment was carried out twice for a slightly different exposure time and a slightly different grating length (by modifying the distance between the mask and the fiber). The spectra in Figs. 9 and 11 show the detailed wavelength dependence in the grating bandwidth and the gradual loss of the individual Fabry–Perot spectrum along the propagation while acquiring the many and random Fabry–Perot characteristics.

The transmitted power measurements shown in Figs. 11 and 12 were done at the center of the grating spectrum
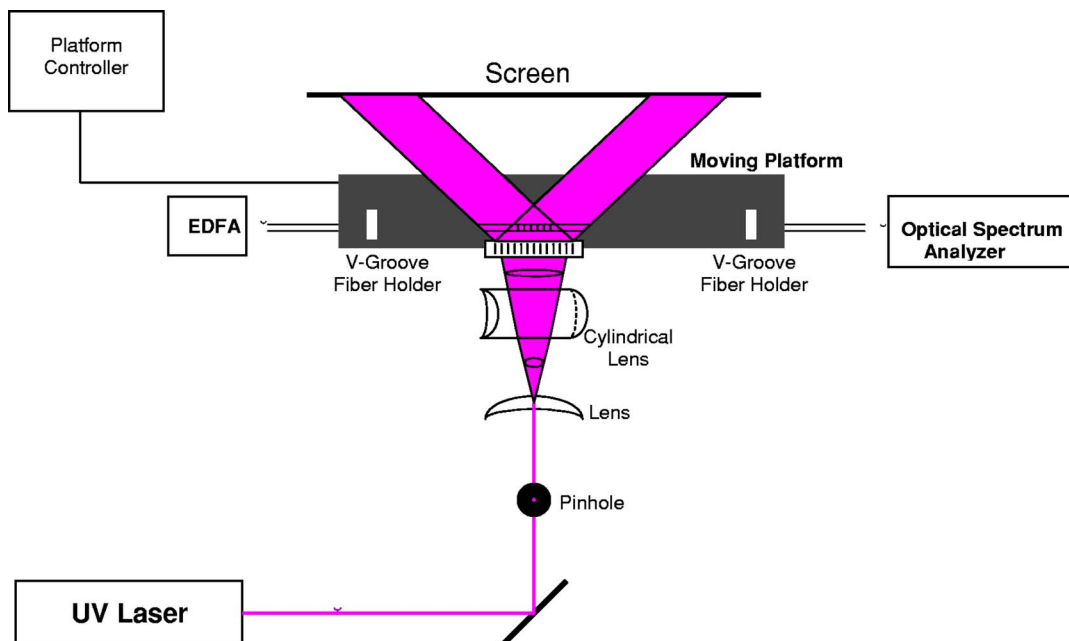

Fig. 8.   (Color online) Experimental setup. The UV laser beam, aligned to a pinhole, is broadened by a lens in order to achieve a larger spot size for the grating writing. The beam is then focused at the fiber axes by a cylindrical lens to obtain maximum intensity on the fiber. A phase-grating diffraction gives two first-order waves, causing a sinusoidal interfering pattern on the fiber. The light transmission measurements in the fiber were done *in situ* after each additional grating fabrication. This procedure allowed us to follow the localization buildup with the grating number.
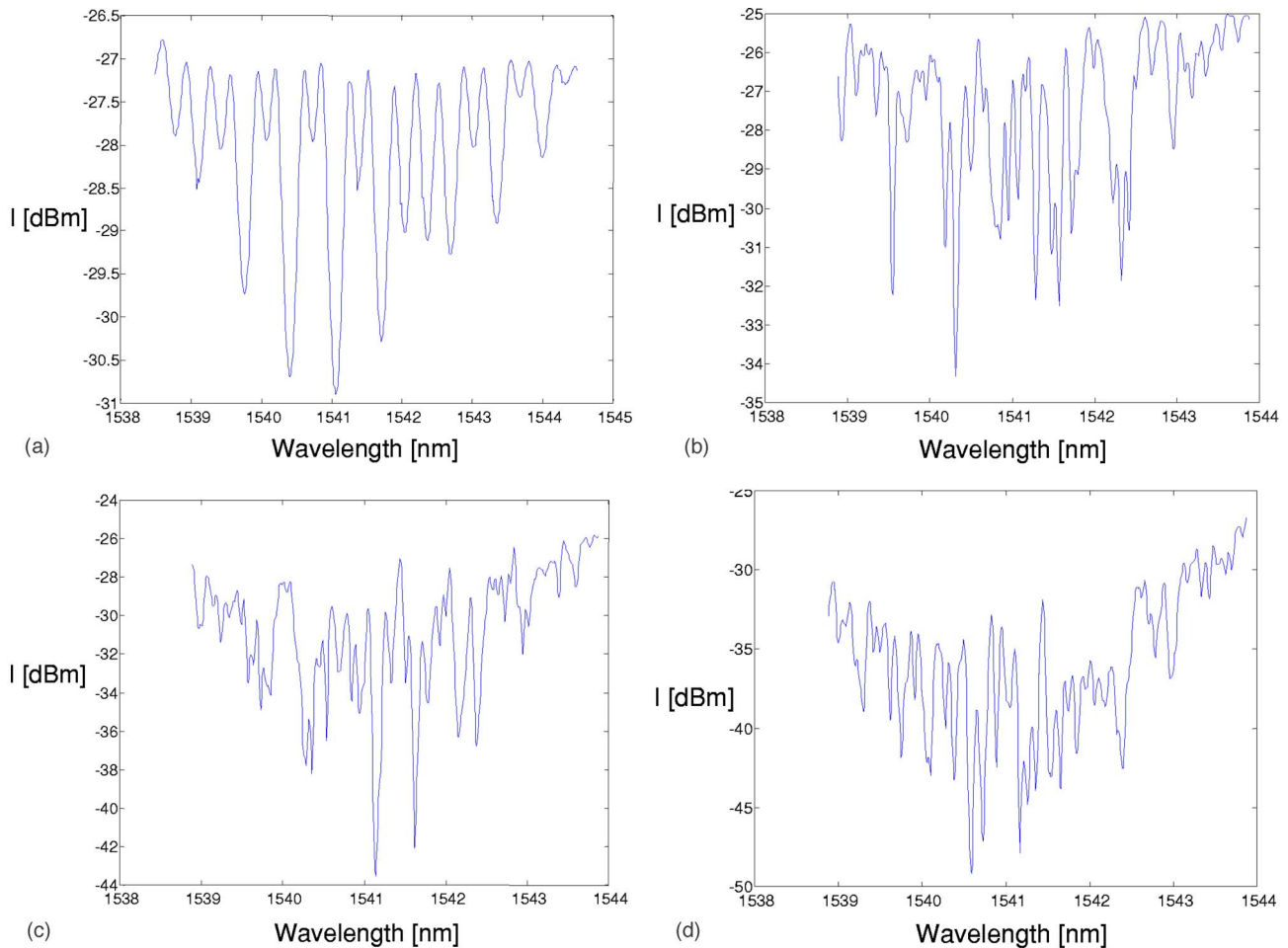
Fig. 9.   (Color online) Experiment 1. Transmitted spectrum measured after (a) 3, (b) 10, (c) 25, and (d) 50 gratings.

band, where the transmission is minimal, and were averaged over 0.5 nm in Experiment No. 1 and over 0.3 nm in Experiment No. 2. The reason for this averaging stems from the need to overcome the random fluctuations and the temperature and stress changes experienced by the fiber during the experiment, causing the measured spectrum to drift and vary. The averaging interval, on the one hand, was chosen to be large enough to suppress those environmental changes, but on the other hand, as the transmissivity magnitude varies with wavelength the interval had to be limited to a length at which the maximum difference in transmission could be tolerated. In more explicit terms, if the grating minimum transmissivity is ~0.4 dB, then an accuracy of 1 magnitude less is tolerable. Furthermore, the noise caused by the optical amplifier and the spectrum-analyzer accuracy results in a measured accuracy no better then 0.02 dB.

C. Experiment versus Theory
Section 2 provided the theoretical asymptotic behavior of the transmission with the exponential decay given by $\tau_N = \exp[-N \ln(1/\tau)]$, where τ is the intensity transmission coefficient of a single grating. Therefore, in order to compare the theoretical results with the findings obtained from the experiments, it is necessary to first find τ at the point in the spectrum where the transmission measurements were taken. This single grating value is to be compared with the experimental decay rate of the complete grating-array transmissivity.

Measurement of the transmissivity of a single grating can be performed by one of two methods. The first and most straightforward method is to take the result obtained from measuring the spectrum of the first grating and normalizing it according to the spectrum measured for a grating-free fiber (which is basically the power spectrum of the EDFA). This method has a great disadvantage in that it features a wide inaccuracy caused by a power drift that may occur between the two measurements as a consequence of fiber bending, polarization dependent loss, and an instability of the source power. Whereas the drift caused by these effects is tolerable for most of the experiment when it is suppressed to values of ~0.1 dB by maintaining a relatively constant temperature, a vibration-free environment, and a minimized fiber movement during the experiment this is not the case when measuring the first grating, as the minimum transmission is in mere tenths of decibels. The second method is to measure the transmission of two gratings, which is a form of Fabry–Perot, and extracting from it the transmissivity of a single grating. Although this method is less straightforward, it has a great advantage over the previous one in that all the effects causing the inaccuracy of the previous method are negligible. This is because the measurement is performed
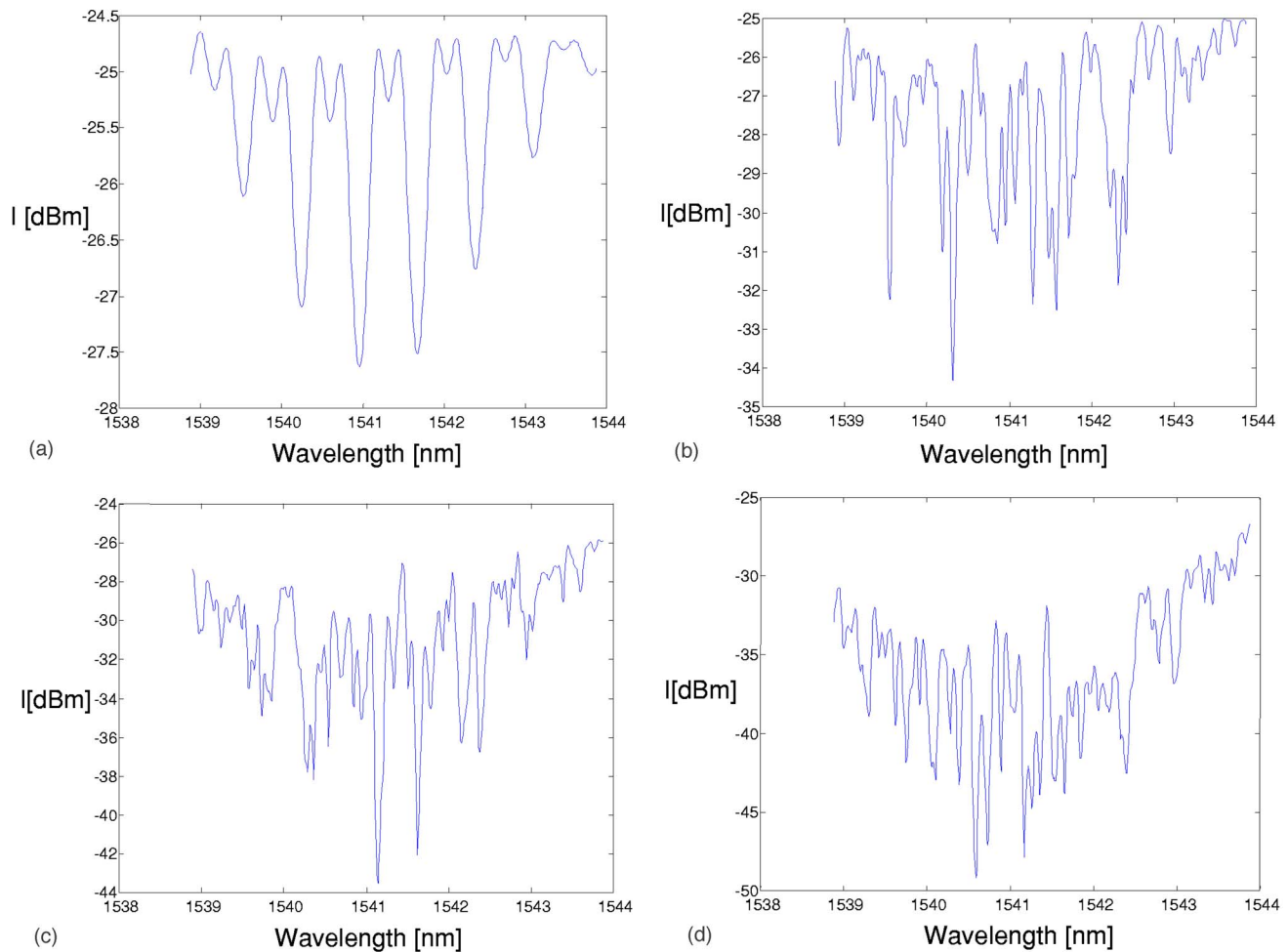
Fig. 10.   (Color online) Experiment 2. Transmitted spectrum measured after (a) 3, (b) 10, (c) 25, and (d) 50 gratings.

at a specific given time without necessitating measurement of the reference level. To achieve good accuracy, the second method was selected. It is now possible to derive the transmissivity of a single grating from the spectrum measurement of two successive gratings. Although the Fabry–Perot properties are simple and known, we derive them here by using the transfer-matrixmethod. The transfer matrix of two successive gratings with a spacing d between them is

$$M_2 = \begin{pmatrix} \dfrac{1}{t^*} & -\dfrac{r^*}{t^*} \\ -\dfrac{r}{t} & \dfrac{1}{t} \end{pmatrix} \begin{bmatrix} \exp(ikd) & 0 \\ 0 & \exp(ikd) \end{bmatrix} \begin{pmatrix} \dfrac{1}{t^*} & -\dfrac{r^*}{t^*} \\ -\dfrac{r}{t} & \dfrac{1}{t} \end{pmatrix},$$

(22)

where t_1 and r_1 are the single-grating amplitude-transmission-reflection coefficients, respectively. The distance between the two gratings is d, and the propagated field wave number is k. Then, the intensity transmissivity of the grating pair can be calculated from $1/T_2=[M_2]_{22}$, which together with $\tau=|t|^2$, $\rho=|r|^2$, $\tau=|t|^2$, $\tau_2=|T_2|^2$ and $\tau+\rho=1$, $\alpha=kd$, gives the simple Fabry–Perot intensity transmissivity $\tau_2=\tau^2/(\tau^2+4\rho\sin^2\delta)$, where $(\delta= 4\pi)nd/\lambda$,

n is the refractive index and λ is the wavelength. Obviously, it is difficult to extract τ from the last expression, as it requires knowledge of the transmission coefficient phase and the exact distance between the gratings. However, we can easily find it from the minimum of τ, as its value does not depend on the phase $\tau_{\min}= \tau^2/(2-\tau)^2$. Then the transmission coefficient of a single grating can be written as $\tau=2\tau_{\min}^{1/2}/(1+\tau_{\min}^{1/2})$.

The grating pair measurements of the normalized transmission spectra are shown in Figs. 13 and 14 for the first two gratings from Experiments 1 and 2, respectively. The value $\tau_{\min}$ is the square root of the minimum transmissivity at the center of the grating spectrum; it is −1.63 dB for Experiment 1 and −1.41 dB for Experiment 2. For Experiment 1, according to Eq. (22), the transmissivity of a single grating is −0.43 dB; for Experiment 2, it is −0.35 dB. According to the localization theory these values are to be compared with the overall transmission slopes that are a−0.405 dB/grating in Experiment 1 and a −0.326 dB/grating for Experiment 2.

D. Loss
Throughout the study, it was assumed that loss is negligible. This assumption should be confirmed experimentally, since although the loss of the fiber itself for such

short distances is negligible, the process of grating fabrication might introduce some additional loss in the grating formation with the exposure to the UV radiation that changes the fiber uniformity and the absorption coeffi-
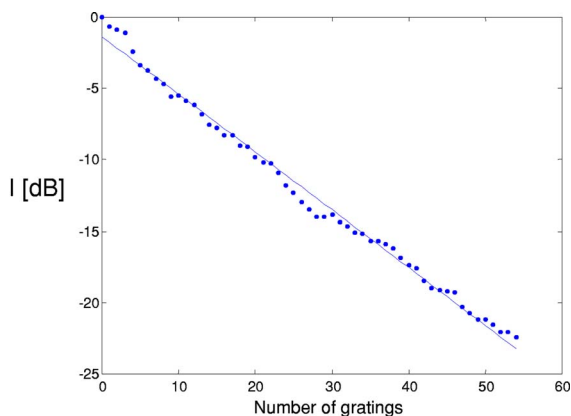

Fig. 11. (Color online) Experiment 1. Transmission measured at the grating center wavelength (at minimum transmission). The fitted straight-line slope is −0.405 dB/grating.
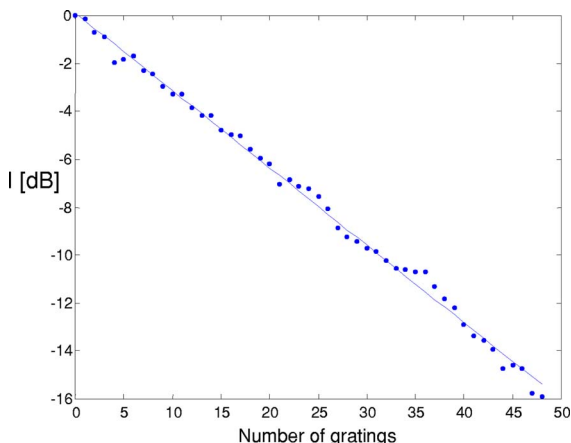

Fig. 12. (Color online) Experiment 2. Transmission measured at the grating center wavelength (at minimum transmission). The fitted straight-line slope is −0.326 dB/grating.
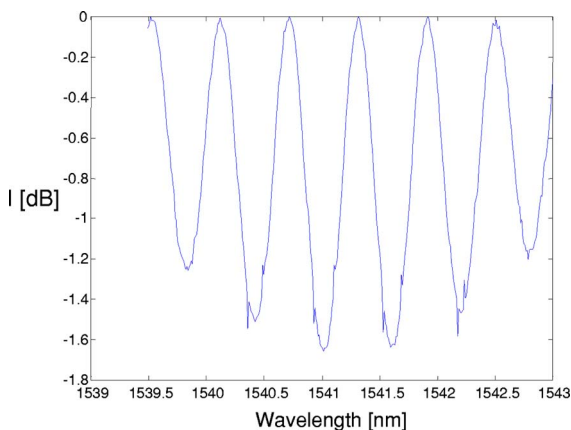

Fig. 13. (Color online) Experiment 1. The normalized power as measured after two gratings is similar to the spectrum of a Fabry–Perot resonator but with the envelope of the grating spectrum. The transmissivity of a single grating is obtained from the ratio between the maximum and the minimum transmission power, which in this experiment resulted in −0.43 dB.
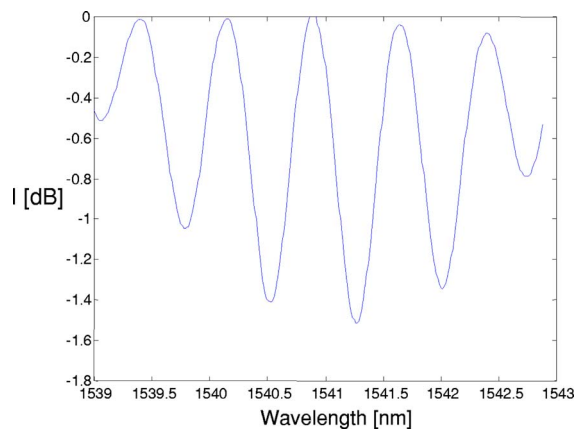

Fig. 14. (Color online) Experiment 2. The transmissivity of a single grating is obtained from the ratio between the maximum and the minimum transmission power, which in this experiment resulted in −0.35 dB.
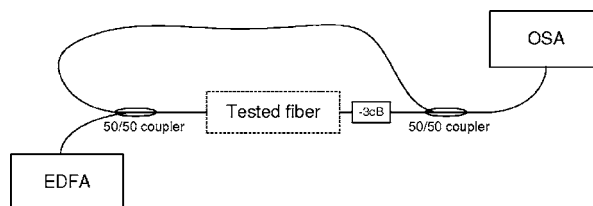

Fig. 15. (Color online) Experiment setup for fiber loss measurement. Light reflected from the tested fiber is rerouted back to the left coupler. Half of the reflection is then present at the left tap of the right coupler and is coupled to half of the transmission from the tested fiber. The right tap of the right coupler is connected to the spectrum analyzer and measures one-quarter of the reflection plus the transmission. The EDFA serves as the light source, and OSA is the optical spectrum analyzer.

cient. To confirm that the exponential decay did indeed result from localization and not from loss, the fiber loss was measured after the grating fabrication. The experiment setup is given in Fig. 15, where one end of the tested fiber was connected to a spectrum analyzer through a 3 dB attenuator and a 50% coupler, and the other end to the EDFA through a 50% coupler. The intensity measured at the coupler output tap was one-quarter of the reflection from the tested fiber plus one-quarter of the transmission. To evaluate the loss, a measurement was made of the spectrum at the coupler-output tap using a regular fiber and then repeated using the tested fiber. Assuming that a regular fiber can be used as a reference for a loss-free fiber, the maximum measured difference between the spectrum of the regular fiber and that of the tested fiber was less then 0.5 dB for all wavelengths. As the total trans-

Table 1. Summary of Experiment Results

Experiment No.	Transmissivity of a Single Grating (dB)	Total Transmissivity Slope (dB/grating)
1	−0.43	−0.405
2	−0.35	−0.326

mission measured was ~ -20 dBm, it may be deduced that the fabricated-fiber loss is negligible.

E. Experimental Conclusion

The overall results show a good agreement between the experimental results and the theory; see Table 1.

The slight difference between the transmission of a single grating and the total transmission slope has four causes. First, the gratings are not exactly equivalent, and therefore the transmission of a single grating (as measured from the first two gratings) can deviate from the average transmission of the gratings. Second, the transmission slope is a consequence of a calculation that evaluates the asymptotic behavior of the transmission. The transmissivity fluctuates in a finite system, deviating from the asymptotic calculation. Third, measurement uncertainties exist, such as temperature, fiber stress, and polarization-dependent loss. Finally, the stability of the EDFA plays a role.

4. SUMMARY

We have presented a realization of Anderson localization with light propagating in one-dimensional randomly spaced gratings in a single mode fiber. We described the theoretical analysis and experimentally demonstrated the localization effect. We measured the transmissivity with the exponential decay along the disordered fiber gratings. The magnitude of the decay rate, i.e., the inverse localization length, is equal to the log of the inverse single-grating transmissivity. The total transmission is comprised of only the wave that passes through all gratings without experiencing any reflections. All other transmitted waves interfere destructively for the transmission. We discussed a ray approach that treats the waves as incoherent owing to an averaging over random phases in the disordered array, but the approach fails to adequately describe the special wave-interference nature.

We conclude with the application sides of the random-grating array. We refer to the reflection side, complementary to the transmission, that can become very large with the strong localization effect. Ordered gratings with their filtering and reflection capabilities are widely used in fiber optics. However, it is very difficult to fabricate gratings longer than a few centimeters. Random-grating arrays are by far easier to make, with much larger lengths. Here, the random nature becomes an advantage. However, even most important feature is that the random array can easily provide very large bandwidth reflection, since it depends on the single-grating bandwidth that can be made very short, thus providing very large wavelength bandwidths of tens of nanometers. Another interesting possibility is the use of the array for fiber lasers. The random grating can provide the pseudocavity for feedback, thus providing a kind of 1D random laser.

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