Simplified Temporal Imaging Systems for Optical Waveforms

José Azaña, Naum K. Berger, Boris Levit, and Baruch Fischer

***Abstract*—We demonstrate that temporal imaging (TI) of optical pulses (distortionless compression or expansion of the optical temporal waveform) can be achieved with a system comprising a quadratic phase modulator (time lens) followed by a dispersive device. As compared with the conventional solution, the proposed configuration does not require the use of an input dispersive device preceding the time lens, thus resulting in a much simpler and more practical alternative for implementing TI of optical signals.**

***Index Terms*—Dispersion, optical pulse propagation, optical pulse shaping, phase modulation, pulse characterization, temporal imaging (TI), time lens.**

TEMPORAL imaging (TI) allows an arbitrarily shaped temporal waveform to be magnified or compressed in time while preserving the shape of its temporal profile [1]–[4]. Specifically, TI of optical waveforms [1]–[3] can be applied for stretching ultrafast optical events to a time scale that is accessible to ordinary high-speed photodiodes and oscilloscopes (temporal microscopy); for time reversal of optical signals; or for waveform preparation on a long time scale with subsequent temporal compression. Conventionally, a TI system for optical waveforms is constructed by cascading input dispersion, quadratic phase temporal modulation (time lens), and output dispersion, in the appropriate balance. The time lens can be practically implemented using EO phase modulation [1], [5], sum-frequency or difference-frequency generation in a nonlinear crystal [2], [3], [6], or cross-phase modulation in a nonlinear fiber [7]. In this letter, a simplified system configuration for TI of optical signals is proposed and investigated. Specifically, we demonstrate that TI of optical waveforms can be achieved with a system configuration similar to that of a conventional optical pulse compressor (i.e., a time lens followed by a dispersive device).

The idea of a TI system where the input signal is not dispersed before receiving the quadratic phase modulation has been previously demonstrated for the case of input electrical signals [4]. A similar ultrashort pulse diagnostic technique, namely spectro-temporal imaging (STI) of optical waveforms (time-to-frequency conversion), has been also demonstrated using systems where the signal is not dispersed before entering the time lens [5], [6].

Manuscript received December 11, 2003; revised August 25, 2004. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada (NSERC), by the Fonds Québécois de la Recherche sur la Nature et des Technologies (FQRNT), and by the Division for Research Funds of the Israel Ministry of Science.

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Digital Object Identifier 10.1109/LPT.2004.837719

Let us assume a system comprising a time lens followed by dispersion. In what follows, we will assume the optical signals to be spectrally centered at the optical frequency ω_0. A time lens is a phase-only modulator with a modulation function [1]

$$m(t) = \exp(j\phi(t)) \propto \exp\left(j\left[\frac{\ddot{\phi}_t}{2}\right]t^2\right) \tag{1}$$

where $\ddot{\phi}_t = \lfloor \partial^2\phi(t)/\partial t^2 \rfloor_{t=0}$ is the frequency chirp induced by the time lens. The symbol $\propto$ indicates that the two functions are proportional. A dispersive medium can be described as a phase-only filter with a spectral transfer function $H(\omega) \propto \exp[j\Phi(\omega)]$ and an associated temporal impulse response [1]

$$h(t) \propto \exp\left(j\left[\frac{1}{2\ddot{\Phi}_\omega}\right]t^2\right) \tag{2}$$

where $\ddot{\Phi}_\omega = \lfloor -\partial^2\Phi(\omega)/\partial\omega^2 \rfloor_{\omega=0}$ is the first-order dispersion coefficient of the medium. Notice that the variable ω is the base-band frequency variable, i.e., $\omega = \omega_{\rm opt} - \omega_0$, where $\omega_{\rm opt}$ is the optical frequency variable. The output optical signal $b(t)$ from our time lens/dispersion system, when a given optical signal $a(t)$ is input to the system, is

$$\begin{aligned} b(t) &= [a(t)m(t)] * h(t) = \int_{-\infty}^{+\infty} a(\tau)m(\tau)h(t-\tau)d\tau \\ &\propto \exp\left(j\left[\frac{1}{2}\ddot{\Phi}_\omega\right]t^2\right) \\ &\quad \times \int_{-\infty}^{+\infty} a(\tau)\exp\left(j\alpha\tau^2\right)\exp\left(-j\left[\frac{1}{\ddot{\Phi}_\omega}\right]t\tau\right)d\tau \end{aligned} \tag{3a}$$

where the asterisk $*$ denotes convolution and

$$\alpha = \left\lfloor \frac{\ddot{\phi}_t}{2} \right\rfloor + \left\lfloor \frac{1}{2\ddot{\Phi}_\omega} \right\rfloor. \tag{3b}$$

The last integral in (3a) can be solved by considering $\exp(-j\lfloor 1/\ddot{\Phi}_\omega \rfloor t\tau)$ as the kernel of a Fourier transform ($\Im$). We derive that

$$\int_{-\infty}^{+\infty} a(\tau)\exp(j\alpha\tau^2)\exp\left(-j\left[\frac{1}{\ddot{\Phi}_\omega}\right]t\tau\right)d\tau = C\left(\omega = \left[\frac{1}{\ddot{\Phi}_\omega}\right]t\right) \tag{4}$$

where $C(\omega) = \Im\{a(\tau)\exp(ja\tau^2)\}$ can be calculated as

$$C(\omega) \propto A(\omega) * \exp\left(-j\left[\frac{1}{4\alpha}\right]\omega^2\right) = \exp\left(-j\left[\frac{1}{4\alpha}\right]\omega^2\right) \times \int_{\Delta\omega_1} A(\Omega)\exp\left(-j\left[\frac{1}{4\alpha}\right]\Omega^2\right)\exp\left(j\left[\frac{1}{2\alpha}\right]\omega\Omega\right)d\Omega \tag{5}$$

with $A(\omega) = \Im\{a(\tau)\}$. The input pulse $a(\tau)$ is assumed to be confined to a spectral bandwidth $\Delta\omega_1$. If this bandwidth is sufficiently narrow so that

$$|\alpha| \gg \frac{\Delta\omega_1{}^2}{16\pi} \tag{6}$$

then the quadratic phase factor $\exp(-j[1/4\alpha]\Omega^2)$ in (5) can be neglected so that

$$C(\omega) \propto \exp\left(-j\left[\frac{1}{4\alpha}\right]\omega^2\right) a\left(\tau = \left[\frac{1}{2\alpha}\right]\omega\right). \tag{7}$$

Introducing (7) into (4) and the result into (3a), we finally obtain that

$$b(t) \propto \exp\left(j\left[\frac{\ddot{\phi}_t}{2M_t}\right]t^2\right) a\left(\tau = \frac{t}{M_t}\right) \tag{8}$$

where M_t is the temporal image magnification factor

$$M_t = 1 + \ddot{\phi}_t\ddot{\Phi}_\omega. \tag{9}$$

Equation (8) indicates that, under the stated conditions, the optical intensity of the output pulse $|b(t)|^2$ is a compressed or expanded replica of the input average optical intensity $|a(t)|^2$, where the temporal image magnification factor is given by (9). This general form of magnification has also been derived in previous work on simplified TI of electrical signals [4]. Moreover, the TI magnification factor given by (9) is identical to that of a conventional TI system [1]–[3]. The reason for this equivalence is that the development presented here is similar to that in conventional TI [1]–[3], except for the fact that the input dispersion is now ignored and the blurring in the TI (due to the elimination of the input dispersion) is forced to be small by inequality (6).

The essential condition to achieve TI with the proposed configuration is given by inequality (6). Note that this condition implies that the bandwidth of the pulse at the output of the time lens is much larger than the input pulse bandwidth, $\Delta\omega_2 \gg \Delta\omega_1$. For design purposes, inequality (6) can be expressed in a more convenient form as follows:

$$|\ddot{\Phi}_\omega| \ll \frac{8\pi}{\Delta\omega_1^2}|M_t|. \tag{10}$$

When designing a time lens/dispersion system for TI (with a desired magnification factor M_t), the specifications of the system must be fixed according to the following procedure. First, the magnitude of the dispersion coefficient $|\ddot{\Phi}_\omega|$ of the dispersive device must be fixed to satisfy (10), and second, the required chirp for the time lens $\ddot{\phi}_t$ must then be obtained from the expression of the magnification factor M_t (9). At this point it should be mentioned that conditions (6) and (10) are too restrictive in practice, much as the similarly derived Fraunhofer distance in the problem of spatial diffraction is far too strict. Depending on the specific temporal shape to be processed, TI can be achieved even if the given conditions are satisfied in a less-restrictive form, e.g., $|\ddot{\Phi}_\omega| < 8\pi|M_t|/\Delta\omega_1^2$.

The reason why the simplified TI technique works properly on electrical signals is because the bandwidth of these signals is generally much narrower than is required to satisfy the stated condition. However, in the case of optical signals (with much broader bandwidths), it is essential to follow the procedure described above in order to ensure the appropriate design and operation of the TI system.

The general condition for TI [e.g., (10)] depends on the maximum spectral bandwidth $\Delta\omega_1$ expected for the input signals to be processed, or in other words, it depends on the fastest temporal feature δt_1 of the input temporal waveforms. Specifically, an estimation of the fundamental temporal resolution provided by the system ($\delta t_1 \approx 2\pi/\Delta\omega_1$) can be obtained from (10) by simple mathematical manipulations, resulting in $\delta t_1 \approx \sqrt{\pi|\ddot{\Phi}_\omega|/2|M_t|}$. For comparison, the fundamental temporal resolution provided by a TI system or an STI system involving the use of input dispersion is mainly limited by the temporal aperture τ_a of the time lens and can be estimated as $\delta t_1 \approx 2\pi/(\tau_a|\ddot{\phi}_t|)$[1], [7]. In both cases, a larger time lens chirp implies a better resolution but in general, which of the two approaches provides a better performance (e.g., in terms of temporal resolutions) essentially depends on the time lens aperture and desired magnification factor; i.e., this should be evaluated for each particular experiment.

Our theoretical findings have been confirmed by proof-of-concept experiments. We consider the specific case of a temporal microscope (TM), where the TI system is designed to achieve direct and magnified temporal images ($M_t > 1$). In this case, it can be inferred from (9) that the time-lens chirp $\ddot{\phi}_t$ and the dispersion coefficient $\ddot{\Phi}_\omega$ must have the same sign. In this instance, the TI condition in (6) can be rewritten as

$$|\ddot{\phi}_t| + \left(\frac{1}{|\ddot{\Phi}_\omega|}\right) \gg \frac{\Delta\omega_1^2}{8\pi}. \tag{11}$$

Note that if inequality (11) is satisfied for a given dispersion magnitude $|\ddot{\Phi}_\omega|$, then it is necessarily satisfied for any smaller amount of dispersion. Thus, if inequality (11) is satisfied for $|\ddot{\Phi}_\omega| \to \infty$, then we should expect to observe a magnified replica of the input temporal waveform by introducing any amount of dispersion after the time lens. In fact, it can be easily proved that if $|\ddot{\Phi}_\omega| \to \infty$, then the condition (11) reduces to a condition for the time-lens chirp, as follows:

$$|\ddot{\phi}_t| \gg \frac{\Delta\omega_1{}^2}{8\pi}. \tag{12}$$

It was recently demonstrated that condition (12) ensures that the time lens modifies the spectrum of an input optical pulse so that the output spectrum is proportional to the input temporal envelope (STI) [5]. Note that again inequality (12) determines the temporal resolution of our TM system (and more, in general,

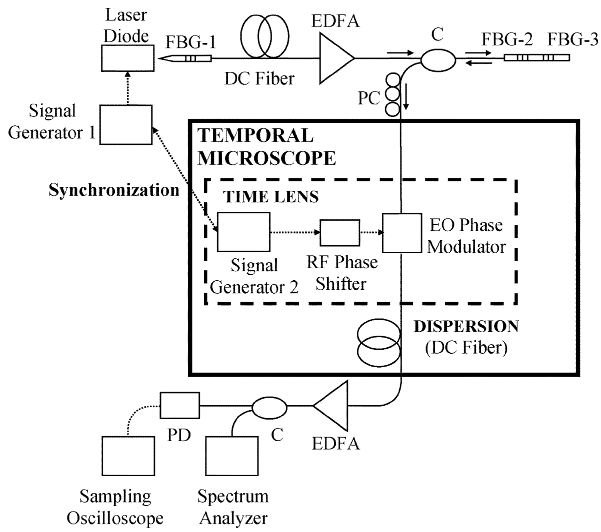

Fig. 1. Schematic of the arrangement used for our proof-of-concept experiments. Solid (dotted) lines are used for optical (electrical) signals. EDFA: Erbium-doped fiber amplifier. C: Fiber coupler. PC: Polarization controller.

it determines the resolution of an STI system based on single time lens [5], [6]), resulting in this case $\delta t_1 \approx \sqrt{\pi/2|\ddot{\phi}_t|}$. The main advantage of our proposal as compared with the conventional configuration for TM of optical waveforms [1]–[3] is that in our system, we only have to ensure that the time lens is strong enough to satisfy *inequality* (12), based on the maximum bandwidth expected for the optical signals to be processed (or desired temporal resolution). In contrast, in the conventional configuration for TM, the three design parameters, namely the chirp of the time lens and the dispersions introduced by the two dispersive lines surrounding the time lens, must be carefully adjusted so that to satisfy exactly the so-called TI condition [1].

Fig. 1 shows a schematic of our experimental setup. Briefly, an actively mode-locked laser diode with an external resonator based on a uniform fiber Bragg grating (FBG-1) and followed by dispersion-compensating (DC) fiber was used as the optical pulse source. This source generated approximately Gaussian optical pulses ($\lambda_0 \approx 1548$ nm) with a full-width at half-maximum (FWHM) time width of ≈17.5 ps. These pulses were subsequently reshaped by means of an FBG-based optical pulse shaper, consisting of two consecutive π-shifted uniform FBGs (FBG-2 and FBG-3), to generate a nonsymmetric double pulse, which was used as the input signal to our TM. A $LiNbO_3$ electrooptic (EO) phase modulator, driven by a sinusoidal radio-frequency (RF) modulation signal, was used as the time lens mechanism [1], [5]. The modulation frequency and modulation index (amplitude) were set to $\omega_m = 2\pi 9.9$ GHz $\cdot$ rad and $A = 2.4$ rad, respectively, thus inducing a chirp of $|\ddot{\phi}_t| \approx A\omega_m{}^2 \approx 9286.27$ GHz$^2 \cdot$ rad [1]. For the dispersive medium following the EO time lens, we used DC optical fiber (positive dispersion coefficient). The half period that it is chosen for the modulation RF signal sign was carefully selected to ensure that $\ddot{\phi}_t$ was also positive. The optical signals were measured in the temporal domain using a fast photodetector (PD) followed by a sampling oscilloscope (bandwidths of ≈50 GHz). Fig. 2 shows the results corresponding to one of our experiments, where we used a section of DC fiber providing a total dispersion

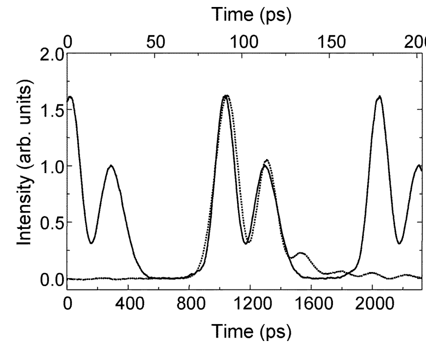

Fig. 2. Dotted curve (top time scale): Measured temporal waveform at the input of the TM. Solid curve (bottom time scale): Measured temporal waveform at the output of the TM. The ratio between the bottom and top time scales is fixed according to the theoretical temporal magnification factor $M_t = +11.45$.

of ≈ -885 ps/nm ($\ddot{\Phi}_\omega \approx 1125$ ps^2/rad). According to (9), a temporal magnification factor of $M_t \approx 11.45$ should be achieved with this configuration. The dotted curve in Fig. 2 shows the measured input temporal waveform (top time scale), which consists of two consecutive and partially overlapped Gaussian optical pulses of different intensity. Each individual pulse has an FWHM time width of ≈17.5 ps and the two pulses are separated by ≈22.5 ps. The observed oscillatory tail is an artifact introduced by the PD. The total bandwidth of the input optical signal (spectrum not shown herein) was estimated to be $\Delta\omega_1 \approx 2\pi 62.5$ GHz$\cdot$rad, so that condition (12) is satisfied in the form $|\ddot{\phi}_t| > \Delta\omega_1{}^2/8\pi$. The solid curve in Fig. 2 shows the measured temporal waveform at the output of our system (bottom time scale). There is an excellent agreement between this output temporal waveform and the temporally magnified replica of the measured input temporal waveform (with $M_t = 11.45$).

We estimate that the TM reported here provides a temporal resolution of ≈10 ps. These numbers are of the order of those achievable with a conventional TI system (assuming that the same time lens is used). In our experiments, we were limited by the chirp magnitude that can be induced with an EO time lens. We estimate that using a similar time lens to that in [2], our TI system could resolve temporal features on the order of a few hundreds of femtoseconds.

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