

# Compression of Periodic Optical Pulses Using Temporal Fractional Talbot Effect

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**Abstract**—Recently, a novel method for compression of periodic optical pulses based on a superposition of replicated and time-delayed original pulse trains was proposed. In this paper, we experimentally demonstrate that the phase shift of these replicas, as occurs in the temporal fractional Talbot effect, leads to compression without repetition-rate multiplication, which is inherent in the previous method. Simple and compact devices based on fiber Bragg gratings or fiber/or waveguide splitters/combiners, used for implementation of the present method can be considered as an equivalent of dispersive delay lines for periodic optical pulses. The proposed devices will also be available, for instance, for pulse generation, pulse-chirp compensation, temporal imaging, and real-time spectrum analysis for periodic optical pulses.

**Index Terms**—Dispersive delay line (DDL), fiber Bragg gratings (FBG), fiber/waveguide splitters/combiners, optical-pulse compression, optical-pulse replication, temporal Talbot effect.

## I. INTRODUCTION

WE HAVE recently reported on a novel method for compression of periodic optical pulses without requiring propagation in a dispersive delay line [1]. In this method, such propagation is replaced by superposition of  $M$  identical, phase modulated, time-delayed replicas of original pulse train. Since all replicas are identical and equidistant, the repetition rate  $f$  of the original pulse train is multiplied by  $M$ . Thus, the compression method in [1] inherently involves pulse repetition-rate multiplication that is not always desirable. If the length of the equivalent dispersive delay line, in which the phase-modulated original pulses would be compressed in the conventional compression method, is equal to the Talbot length  $z_T$  (defined below) for the multiplied repetition rate  $M \cdot f$ , then the pulses obtained are compressed.

In this paper, we propose and experimentally demonstrate another method of periodic pulse compression that does not need a dispersive delay line (in the ordinary sense), but keeps the original repetition rate. The present method is analogously based on a superposition of phase-modulated, replicated and time-delayed pulse trains. However, in this method, the replicas of the phase-modulated original pulse train are phase shifted in a way that resembles the wave behavior in the temporal fractional Talbot effect. The pulse compression occurs if the length of the equivalent dispersive delay line is equal, in contrast to [1], to the fractional Talbot length  $z_{fT}$  (defined below) for the original repetition rate  $f$ . Since the pulse-train replicas in the

present method are not identical, the superimposed pulse trains have the same repetition rate as the original pulse train.

The temporal Talbot effect [2] is the temporal analog of the spatial Talbot effect [3], implying that after propagation of periodic pulses in a dispersive medium, the temporal shape of the pulses is reproduced at multiples of the so-called Talbot length, defined by  $z_T = T^2/\pi|\beta_2|$ , where  $T$  is the pulse period and  $\beta_2$  is the group velocity dispersion. For the fractional temporal Talbot effect [4], [1], the length  $L$  of the dispersive line is chosen to be equal to the fractional Talbot length  $z_{fT} = (m/p)z_T$

$$L = (m/p)T^2/\pi|\beta_2| \quad (1)$$

where  $m$  and  $p$  are integers with no common factor. The field amplitude of the pulses at the line output can be obtained in a similar way as in [5] for the spatial fractional Talbot effect

$$E(t, mz_T/p) = \sum_{n=0}^{p-1} C(n, m, p) E(t - nT/p, 0) \quad (2)$$

where  $E(t, 0)$  is the input field amplitude of the pulses, and the Talbot coefficients  $C(n, m, p)$  are given by

$$C(n, m, p) = (1/p) \sum_{q=0}^{p-1} \exp[2i\pi q/p(n - \beta_2 m q/|\beta_2|)]. \quad (3)$$

It can be seen from (2) that for the fractional Talbot effect the pulse field represents a sum of replicas of the original pulse train, each weighted by the complex factor  $C(n, m, p)$ . The number  $M$  of nonzero replicas in (2) is equal to  $p$  for  $p$  odd or to  $p/2$  for  $p$  even [5]. All the nonzero coefficients  $C(n, m, p)$  have the same absolute values for fixed  $p$ . The time delay and phase shift of the  $n$ th replica are given by  $\Delta t_n = nT/M$  and  $\Delta\varphi_n = \arg[C(n, m, p)]$ , respectively.

## II. PRINCIPLE OF COMPRESSION

In the conventional compression method illustrated in Fig. 1(a), original pulses [solid line in (i)] are phase-modulated [dotted line in (i)] and then propagated along a distance  $L$  in a dispersive delay line, resulting in compressed pulses (ii). Let us assume that the numbers  $m$  and  $p$  as well as the pulse period  $T$  are chosen to meet the condition in (1) for this length  $L$ , which provides optimal compression in the conventional method. In this case, according to (2), the compressed pulses (ii) can be represented as a superposition of replicated, time-delayed, and phase-shifted pulse trains (iii). In the present method [Fig. 1(b) and (c)], such replicas [(ii) in Fig. 1(b)] of the previously phase-modulated pulses with a time delay  $\Delta t_n$  and a phase shift  $\Delta\varphi_n$  are produced by using  $M$

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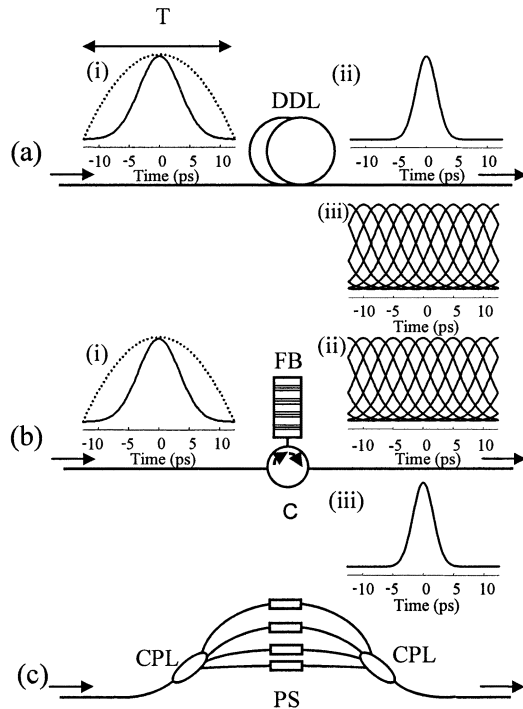


Fig. 1. In the conventional method (a), the original pulse train [solid line in (i)] after quadratic phase modulation (dotted line) is propagated through a dispersive-delay line (DDL), giving the compressed pulses (ii). Under condition (1), these compressed pulses can be represented, according to (2), as a superposition of  $M$  replicas (iii) of the original phase modulated pulse train. In the proposed method [(b) and (c)], the compressed pulses (iii) are obtained by superposition of  $M$  such replicas (ii) of the original phase modulated pulses (i) with the help of (b)  $M$  phase-shifted fiber-Bragg gratings (FBG) or (c) fiber or waveguide couplers (CPL)  $1 \times M$  and  $M \times 1$  with phase shifters (PS).  $C$  is the circulator.

phase-shifted FBG [Fig. 1(b)] or fiber/or waveguide  $1 \times M$  and  $M \times 1$  couplers together with phase shifters [Fig. 1(c)]. The superposition of the replicated pulse trains gives, according to (2), compressed pulses [iii in Fig. 1(b)], which would have been obtained in the conventional method at the output of the equivalent dispersive delay line with the length  $L$  defined by (1). It is important to note that the replicated pulse trains have to overlap in this method, as in [1]. When these replicas do not overlap, the fractional Talbot effect is used for multiplication of the pulse-intensity repetition rate without changes in the pulse shape [4], [6]. Note that the phase shift between  $M$  FBG or  $M$  fiber/waveguide channels is one of the fundamental differences between the methods proposed in [1] and in the present paper. As a result, repetition-rate multiplication does not occur in the present method because the replicas are not identical. The Bragg grating reflectivity should be low in order to prevent multiple reflections.

For example, the following parameters were chosen for the calculation of the pulses in Fig. 1(b):  $T = 25$  ps (only one period is shown),  $m = 1, p = 20$  ( $L = z_T/20, M = 10$ ). In this case, the pulse is compressed from 8 to 4 ps.

It can be proven that the spectral responses of the delay line of length  $L$  and the devices shown in Fig. 1(b) and (c) are identical for the discrete frequencies of pulse-train harmonics  $\omega_0 \pm 2\pi r/T$  ( $\omega_0$  is the central light frequency,  $r$  is an integer),

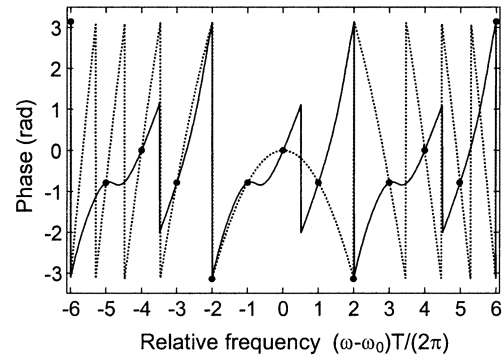


Fig. 2. Calculated spectral-phase responses of a dispersive delay line (dotted line) with the length  $L = z_T/8$  ( $\beta_2 > 0, m = 1, p = 8$ ) and the devices shown in Fig. 1(b) and (c) with  $M = 4$  (solid line). The circles show that these phases are equal for the frequencies  $\omega = \omega_0 \pm 2\pi r/T$ .

if  $L$  and  $T$  meet condition (1). Fig. 2 shows the calculated spectral-phase responses of a DDL (dotted line) with the length  $L = z_T/8$  ( $\beta_2 > 0, m = 1, p = 8$ ) and the devices shown in Fig. 1(b) and (c) (solid line) with  $M = 4$ . It can be seen that the curves intersect at the above-mentioned frequencies. Thus, the devices in the present method act as a phase-only filter, whereas those used in [1] represent an amplitude (as well as phase) filter, rejecting undesirable harmonics. Consequently, the proposed devices can be considered as an equivalent of a DDL for periodic optical pulses. This is also a fundamental difference from the results obtained in [1].

### III. EXPERIMENTAL RESULTS

In the experimental demonstration of the pulse-compression method proposed, we realized a particular case of pulse compression generating the optical pulses from continuous wave (cw) radiation. In the conventional method [7], [8], cw laser radiation is sinusoidally phase modulated and then propagated through a DDL. In our method, the pulse-train replication substitutes for propagation in the DDL. According to the space-time duality [9], a sinusoidal phase modulation acts like a number of "time lenses." The modulation amplitude  $A$  (modulation index) determines the "focal length" of these "time lenses." The modulation index was calculated numerically from the condition that optimal "focusing" would be obtained in the DDL [8] with the dispersion  $\beta_2 L$  determined by (1) {from [8] it follows that  $A \sim 1/(4\pi^2 \beta_2 L f^2)$ }. Generating pulses from cw radiation can be considered as compression of square pulses, having a width  $T$  and period  $T$  [8].

In the case of  $m = 1, p = 8$  ( $L = z_T/8, M = 4$ ), four Bragg gratings should produce, according to (2) and (3) (for  $\beta_2 > 0$ ), four replicas of the original pulse train with a time delay of  $T/4$  between the adjacent replicas and phase shifts of  $-\pi/4, 0, 3\pi/4$ , and  $0$ . We note that in [1], there was no phase shift between the four Bragg gratings.

The radiation of a tunable cw laser diode, sinusoidally phase modulated by a LiNbO<sub>3</sub> electrooptic modulator, was reflected from four FBG, fabricated by a UV laser ( $\lambda = 244$  nm) radiation through a phase mask. The reflectivity magnitude (1.2%) and reflection phase of each grating were controlled by a method similar to that described in [10]. The length of each grating was 0.2

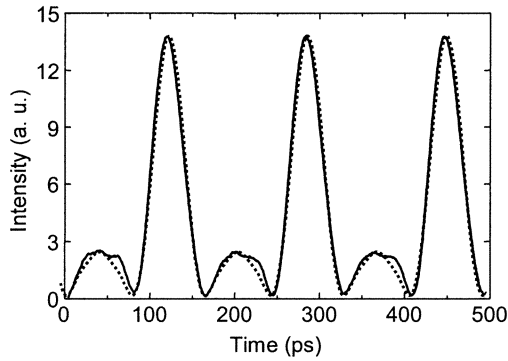


Fig. 3. Experimental pulses (solid line) obtained by reflection of the sinusoidally phase-modulated light from the four phase-shifted Bragg gratings. Calculation (dotted line) of the sinusoidally phase-modulated light transmitted through a DDL with  $L = z_T/8$  ( $DL = -846$  ps/nm). The pulse-repetition rate and the modulation index are 6.1 GHz and 1.4 rad, respectively.

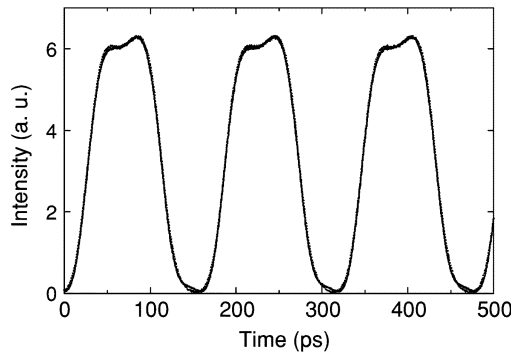


Fig. 4. Experimental pulse trains of the transmission of sinusoidally phase-modulated light via two systems (the curves almost coincide): the asymmetrical Mach-Zehnder interferometer (solid line) and the fiber (dotted line) with a length of  $L = z_T/4$  ( $DL = -1627$  ps/nm). The pulse repetition rate and the modulation index are 6.25 GHz and 0.8 rad, respectively.

mm, enabling an almost constant reflectivity in the region of the input-pulse spectrum. The distance between the gratings was 4 mm. Fig. 3 shows the experimental pulses (solid curve) obtained by reflection of the sinusoidally phase-modulated light from the four Bragg gratings with appropriate phase shifts for the modulation frequency 6.1 GHz and modulation index 1.4 rad. For comparison, the dotted curve in Fig. 3 shows the calculation for transmission of the same phase-modulated radiation through the equivalent DDL with dispersion of  $DL = -846$  ps/nm ( $D = -2\pi c\beta_2/\lambda^2$ ,  $c$  is the velocity of light and  $\lambda$  is the central wavelength), corresponded to the condition  $L = z_T/8$  (for wavelength  $\lambda = 1543.2$  nm). The excellent agreement of these curves proves that the four uniform phase-shifted Bragg gratings act in this case as a DDL with dispersion of  $DL = -846$  ps/nm. Such phase-shifted gratings, in contrast to four identical FBG in [1], do not give pulse-rate multiplication (compare Fig. 3 in the present paper and Figs. 3 and 6 in [1]).

In the experiment for the case of  $m = 1, p = 4$  ( $L = z_T/4, M = 2$ ), sinusoidally phase-modulated light was transmitted through a device similar to the one shown in Fig. 1(c),

but with two channels (all-fiber asymmetrical Mach-Zehnder interferometer with two couplers 50/50). The fiber length difference of the two channels was approximately 16 mm. Exact values of the time delay  $T/2$  between the channels and the phase shift  $\pi/2$  (according to (2) and (3)) between the replicated pulse trains were adjusted by tuning the modulation frequency and the laser wavelength, respectively. Fig. 4 shows, for comparison, the experimental pulses obtained after transmission of sinusoidally phase-modulated light via the Mach-Zehnder interferometer (solid line) and the fiber with the total dispersion  $DL = -1627$  ps/nm (dotted line) that corresponds to the condition  $L = z_T/4$  for the pulse rate 6.25 GHz. The modulation index was 0.8 rad. As can be seen in Fig. 4, the agreement between the results is excellent (the curves almost coincide). In this case, the asymmetrical Mach-Zehnder interferometer is equivalent to the DDL with a dispersion of  $DL = -1627$  ps/nm.

#### IV. CONCLUSION

It is important to note that the devices based on couplers have a broadband spectral response and, therefore, can be especially useful for compression of ultrashort pulses. The proposed devices can also be used as an equivalent of DDLs, for instance, for pulse generation, pulse-chirp compensation, temporal imaging [9], and real-time spectrum analysis [11], [12] for periodic optical pulses.

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