Evolution of localization in frequency for modulated light pulses in a recirculating fiber loop

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We present an experimental demonstration of the evolution of localization in frequency of light pulses that are repeatedly kicked by phase modulation and then propagated along equally spaced lengths of fiber with weak dispersion. The experiment was performed with a long fiber recirculating loop that allows us to follow the pulse's spectral changes after each cycle.

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Light pulses in a dispersive medium that undergo repeated phase modulation upon propagation at equal distances were shown to behave as quantum-kicked rotors[1] with localization properties in their frequency domains. This behavior is related to Anderson localization for electrons in one-dimensional disordered solids.[2,3] The long-term localization behavior in the kicked optical system was demonstrated with mode-locked dispersive fiber lasers.[4] These lasers were shown to exhibit confined exponential spectra, which are typical of localization, besides special resonances (dispersion modes) in some regions. As in many other cases, such experimental studies skip the buildup step that commonly endures a few kicks and is in general difficult to follow. Study of this buildup stage is our aim in the present research.

We present an experimental demonstration of the evolution of localization in frequency of the optical kicked rotor in dispersive single-mode fibers. The experiment was performed with a long recirculating fiber loop such that we could follow every round trip of propagation of the light in the loop. The optical system provides a unique opportunity to track the buildup of localization, almost an impossible task in the usual quantum-kicked rotor. This system also provides the opportunity for following the buildup of pulses in the time and frequency domains.

The localization received here occurs after propagation in a dispersive fiber of broad light pulses that are repeatedly kicked by sinusoidal phase modulation at equally spaced locations along the fiber. The naïve expectation concerning the evolution of the spectrum and the buildup of sidebands (harmonics) is that their number will diffusively increase with the number of kicks, such that the spectrum will continually broaden with propagation. However, because of localization the spectrum is confined, usually with an exponential signature. The focus of this Letter is on the transition between the broadening and the localization regimes and on the number of kicks needed for it to occur. The experiment was performed with a recirculating fiber loop system that enabled the pulse to be tracked after each round trip of the loop.

The electric-field amplitude ψ of a pulse that is propagating in dispersive single-mode fibers, in the slowly varying amplitude approximation, satisfies a normalized Schrödinger-like equation, with a potential that results from the periodic modulation (kicks):

$$i\,\frac{\partial\psi}{\partial z^*} = \gamma\,\frac{\partial^2\psi}{\partial T^2} + A\cos T\sum_N \delta(z^* - N)\psi\,, \qquad (1)$$

where N is the number of kicks, $z^* = z/z_0$ is the spatial propagation coordinate normalized to the length of cavity z_0, $T = \Omega\tau$ is the internal time variable relative to the center of the pulse and multiplied by modulation frequency Ω, and $\gamma = {}^1\!/_2\,\beta_2 z_0 \Omega^2$. We do not include absorption in Eq. (1), as it can be compensated for by an amplifier.

For weak dispersion, $\gamma \ll \pi$, we can write the standard discrete time mapping for the optical kicked rotor as

$$p_N = p_0 - \sum_{M=1}^{N} \kappa \sin T_M\,,$$
$$T_N = T_0 + \sum_{M=0}^{N-1} p_M\,, \qquad (2)$$

where $p_N = 2\gamma n_N$ is the angular momentum, $n_N = n|_{z^*=N^+}$ is the sideband number, $T_N = T|_{z^*=N^+}$, and $\kappa = 2\gamma A$.

It can be seen that from Eqs. (2) we can obtain the following relation for p_N:

$$p_{N+1} - p_{N-1} = 2\kappa\sin(T_N + p_N/2)\cos(p_N/2)\,. \qquad (3)$$

When p_N is an odd product of π, the changes to p cancel each other out, resulting in a classical barrier. Otherwise, p is uniform (at resonance). We can find the border between the uniform spectrum section and the classical barrier by first approximating Eqs. (2) naïvely:

$$p_N \approx p_0 - N\kappa\sin T_0 - {}^1\!/_2\,N^2\kappa p_0\cos T_0,$$
$$T_N \approx T_0 + Np_0\,. \qquad (4)$$

This approximation is valid only when the following assumptions hold:

$$Np_0 \ll \pi\,, \tag{5}$$

$$N\kappa \ll p_0\,. \tag{6}$$

However, because of the dependence of the assumptions on N, after enough kicks the assumptions will always fail. When inequality (6) fails first, relative changes in p appear before T changes. If p can change freely, then p_0 is in the uniform spectrum section. When inequality (5) fails first, the changes in p are canceled before any relative changes occur; thus p_0 is in the classical barrier section. The border between these sections is established when both of the assumptions fail together, or $2\gamma\Delta n = p_0 \sim \sqrt{\kappa}$. This gives us the number of sidebands:

$$\Delta n \sim \sqrt{A/\gamma}\,. \tag{7}$$

When we combine inequalities (5) and (6) and require that they fail together, we receive an approximation for the number of kicks required for the transition from spectral broadening to localization to occur:

$$N^2\kappa \sim 1 \Rightarrow N \sim \sqrt{1/\kappa} \sim \sqrt{1/\gamma}\,. \tag{8}$$

The experimental system, shown schematically in Fig. 1, consisted of a recirculating loop composed of an optical fiber, an erbium-doped fiber amplifier, a $LiNbO_3$ phase modulator, a chirped fiber Bragg grating, polarization controls, and an electro-optic switch to control the input and output of the light in the loop. The input to the system was a light pulse with a low repetition rate obtained by a $LiNbO_3$ amplitude modulator. We used 3 km of dispersion-shifted fiber (DSF; $\beta_2 \approx 1\ \mathrm{ps}^2/\mathrm{km}$ for $\lambda = 1550$ nm). The purpose of using the filter was to minimize the possibility of the system's lasing and consisted of a circulator along with a chirped fiber Bragg grating. The electro-optic switch allowed the loop to be opened or closed; when the loop was open, the input pulse entered the loop and the circulating pulse exited the loop, and when the loop was closed there was a broad pulse circulating in it.

For high dispersion (large γ) localization occurs almost immediately, so to observe the evolution as opposed to the long-term behavior of localization we operated near the resonance regime, where $\gamma \ll 1$. The total dispersion of one round trip of the loop is approximated as $\beta_2 z_0 \approx -12\ \mathrm{ps}^2$, determined by the fiber and the chirp of the Bragg grating, leaving γ dependent only on the frequency. We require that the phase modulation be synchronized to the phase of the pulses propagating in the loop. Otherwise we receive destructive interference among the different modes, so no localization of the spectrum will occur. In addition, operation at modulation frequencies that correspond to the Talbot length or to fractions of the Talbot length will result not in localization but rather in good-quality pulses after mode locking is achieved.[5]

In Figs. 2 and 3 we present experimental results and a numerical simulation for the evolution of the localization. In Fig. 2 spectra can be seen for different numbers of kicks, where $f = 4.5$ GHz and $\gamma \approx 0.0048$. It can be seen that there is spectral broadening for lower kicks and that for higher kicks the spectral width remains the same, whereas the different sidebands have different intensities, which we observed to be periodic. The spectral envelope is not exponential, as is expected for localization behavior, because of operation near resonance.[6,7] At resonance the dispersion is effectively eliminated, leaving only modulation, which causes broadening of the spectrum.

Figure 3 shows the simulation and the experimental results, where the spectral width for each kick was calculated with the average standard deviation and then represented as a function of the number of kicks. The experimental results versus numerical simulation of the localization buildup can be observed for $f = 4.5$ GHz and $f = 7$ GHz (where $\gamma \approx 0.0048$, and $\gamma \approx 0.011$ respectively), where there is diffusive broadening and then confinement. The localization

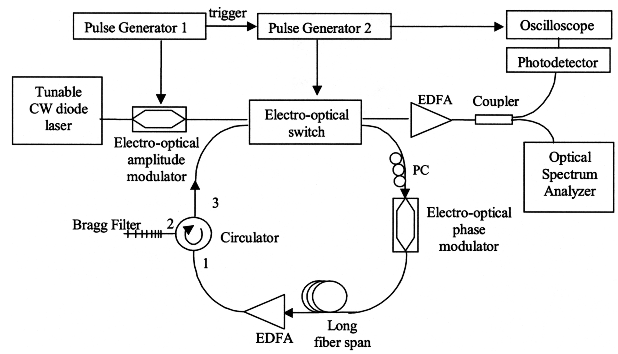

Fig. 1. Schematic of experimental system consisting of optical fiber, $LiNbO_3$ phase and amplitude modulators, polarization controllers (PCs), a circulator, a fiber Bragg grating, erbium-doped fiber amplifiers (EDFAs), a tunable-diode laser, and an electro-optic switch.

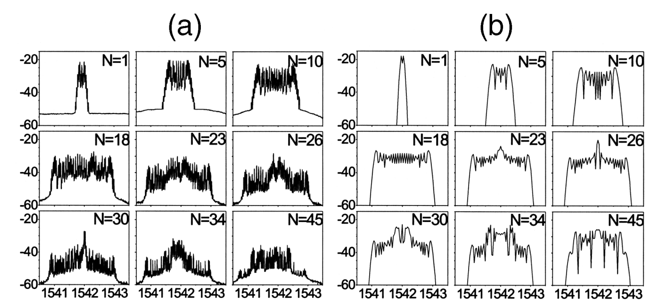

Fig. 2. Spectra obtained from (a) the experiment and (b) a numerical simulation for various numbers of kicks (N), showing localization in frequency for $L = 3$ km DSF, where $f = 4.5$ GHz and $\gamma \approx 0.0048$. The axis is the power [dBm] versus wavelength [nm] in all cases.

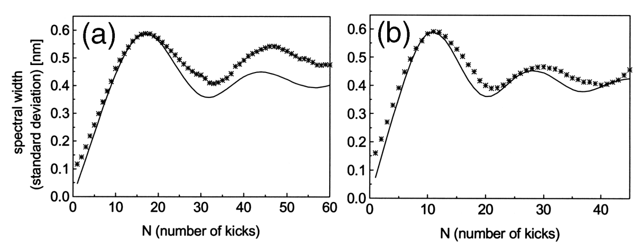

Fig. 3. Evolution of localization for $L = 3$ km DSF, where (a) $f = 4.5$ GHz, $\gamma \approx 0.0048$ and (b) $f = 7$ GHz, $\gamma \approx 0.011$; *, experimental values; solid curves, numerical simulation.

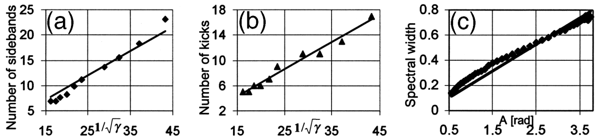

Fig. 4. Experimental results for (a) the number of sidebands and (b) the number of kicks for the transition to localization, both as functions of $\sqrt{1/\gamma}$; (c) spectral width as a function of modulation amplitude, before localization, for $f = 4$ GHz ($\gamma \approx 0.0038$). In all cases, $L = 3$ km DSF.

is characterized by oscillations about some average spectral width. It can be seen that localization occurs sooner for larger γ. The agreement between the experimental and the theoretical results is very good.

To verify the linear relations presented in expressions (7) and (8) we performed a series of experiments in which the cavity length and the modulation amplitude remained constant. The only change was in the modulation frequency, which varies the value of γ. The frequencies were varied from 3 to 8 GHz, in steps of 0.5 GHz, except at 5 GHz. For each γ, the spectrum was measured for every kick as long as measurement was possible (until lasing of the system or loss of synchronization). The results calculated from the experimental data are presented in Fig. 4. The number of sidebands was calculated as $\Delta n = B/\Omega$, where B is the average spectral width of the localization. It can be seen [Fig. 4(a)] that there is good agreement between the theory and the experiment; the experimental inaccuracies for small γ can be explained as being due to an insufficient number of kicks (spectral widths) being available for averaging. As for large γ, the linear relationship is expected to fail (assumptions made are no longer valid), as can be observed in the deviation from linearity as $\sqrt{1/\gamma}$ goes to zero.

In considering the relationship between the number of kicks necessary for localization to $\sqrt{1/\gamma}$ [Fig. 4(b)], the criteria that we used was selection of the kick found by intersection between the spectral broadening region and the average spectral width, defined above. This average width represents the long-term localization behavior, meaning that overall localization actually occurs when this average is reached and that any additional spectral broadening is due to oscillations. Here, also, the averaging is inaccurate.

It can be seen that there is good correlation between the results and the theory. However, note that the linear line does not intersect the zero; there is a shift of two kicks. This shows that the criterion used is not exact.

It can be deduced from Fig. 4(c) that Δn should also be linear with $\sqrt{A}$ for a specific γ. To verify this experimentally, γ and the number of kicks must be kept constant and the parameter varied is A, the modulation amplitude. Verifying this relationship proved to be impossible in the experiment because of the large oscillations about the average spectral width, which are characteristic of the localization behavior. These oscillations depend on γ but also on A, thus producing too many fluctuations in the number of sidebands.

The dependence of Δn on $\sqrt{A}$ in the localization regime [relation (7)] could not be verified experimentally because of the large oscillations that depend on A. However, before localization occurs, we achieve spectral broadening without oscillations; thus the dependence of n on A is linear according to the classical theory. For a small number of kicks, the assumptions made in approximations (4) are valid, and it can be seen that the angular momentum is proportional to κ and that the spectral width is proportional to A:

$$2\gamma\Delta n \sim \kappa \Rightarrow B \sim \Omega\kappa \sim A. \tag{9}$$

This linear relationship was verified in the experiment, as shown in Fig. 4(c): The number of kicks (loops) was 10 (before localization occurred), the fiber was the same as before, $f = 4$ GHz, and $\gamma \approx 0.0038$. The linear relationship is clear; the slight deviations are a result of the instability of the experimental system and the small shift of the broadening behavior as a function of A. The linear relationship can be seen to fail for large A, where it cannot be assumed that we are in the region before localization.

In conclusion, we have presented experimental verification of the evolution of localization in optical fibers and shown additional behavior for weak dispersion.

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References

1. B. Fischer, A. Rosen, and S. Fishman, Opt. Lett. **24,** 1463 (1999).
2. P. W. Anderson, Phys. Rev. **109,** 1492 (1958).
3. For reviews see D. J. Thouless, in *Critical Phenomena, Random Systems, Gauge Theories*, K. Osterwalder and R. Stora, eds., Proceedings of the Les-Houches summer school (North-Holland, Amsterdam, 1986), p. 681.
4. B. Fischer, B. Vodonos, S. Atkins, and A. Bekker, Opt. Lett. **27,** 1061 (2002).
5. B. Fischer, B. Vodonos, S. Atkins, and A. Bekker, Opt. Lett. **25,** 728 (2000).
6. B. Fischer, A. Rosen, A. Bekker, and S. Fishman, Phys. Rev. E **61,** R4694 (2000).
7. A. Rosen, B. Fischer, A. Bekker, and S. Fishman, J. Opt. Soc. Am. B **17,** 1579 (2000).