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# Compression of periodic light pulses using all-optical repetition rate multiplication

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**Abstract**

We propose a novel method for compression of periodic optical pulses based on all-optical repetition rate multiplication of pulses without requiring propagation in a dispersive delay line. The compression principle is explained using the temporal Talbot effect. The proposed method is demonstrated experimentally with the generation of ~20 ps pulses from cw radiation of a laser diode. The repetition rate multiplication is performed with fiber Bragg gratings. The proposed method simultaneously implements two important requirements of many fields, for example, of optical communications: pulse compression and pulse repetition rate multiplication.


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*Keywords:* Pulse rate multiplication; Pulse compression; Temporal Talbot effect; Fiber Bragg grating

Conventional light pulse compression is accomplished in two stages, first by performing quadratic phase modulation of the pulses (linear chirp of frequency) and then propagating the modulated pulses through a dispersive delay line [1]. In this paper, we demonstrate a novel method of pulse compression of periodic optical pulses that does not require propagation of the pulses in a dispersive delay line. This method is based on the use of all-optical pulse repetition rate multiplication. The proposed technique allows implementation of two important operations in pulse processing, namely pulse compression and repetition rate multiplication, using a simple and compact device.

In the present method the original pulses ($I_0$ in Fig. 1(a)) are phase modulated ($\varphi$ in Fig. 1(a)) as in the conventional compression method, and then replicated and shifted $M-1$ times by an amount of $T/M$ each time (where $T = 1/f$ is the pulse period), resulting in a rate multiplication (by $M$) of the pulse train ($I_{\mathrm{mult}}$ in Fig. 1(a)). This multiplication can be performed, for instance, with the help

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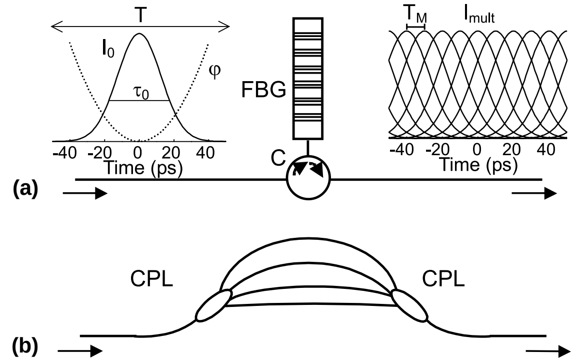


Fig. 1. Methods of all-optical pulse rate multiplication by $M$: (a) using $M$ low reflecting fiber Bragg gratings; (b) using fiber or waveguide couplers $1 \times M$ and $M \times 1$. $I_0$ are the original pulses with period $T$ and $I_{\mathrm{mult}}$ are the multiplied pulses with period $T_{\mathrm{M}}$ (phase modulation of the multiplied pulses is not shown). FBG are fiber Bragg gratings, C is a circulator, CPL is a coupler.

of $M$ fiber Bragg gratings (Fig. 1(a)) or fiber or waveguide couplers $1 \times M$ and $M \times 1$ (Fig. 1(b)). (Reflection from each grating should be small to prevent multiple reflections.) It will be shown later that under certain conditions the superposition and interference of the multiplied pulses (taking into account their phases) results in multiplied compressed pulses.

First, we shall restrict our consideration to Gaussian pulses with quadratic phase modulation. The train of the multiplied pulses can be represented as an expansion in a Fourier series

$$E_{\mathrm{mult}}(t) = \sum_{n=-\infty}^{\infty} c_n \exp(\mathrm{i}\theta_n) \exp(\mathrm{i}2\pi nt/T_{\mathrm{M}}), \qquad (1)$$

where $E_{\mathrm{mult}}(t)$ and $T_{\mathrm{M}}$ are the field and period, respectively, of the multiplied pulses ($T_{\mathrm{M}} = T/M$), $c_n$ and $\theta_n$ are the absolute value and argument, respectively, of $F(2\pi n/T_{\mathrm{M}})$ and $F(\omega)$ is the Fourier transform of the field of the single phase modulated pulse in the train. Let us assume that these phase modulated and multiplied pulses can be compressed by a dispersive delay line as in the conventional compression method. In this case, the spectral phase $\theta_n$ is compensated by the phase $2\pi^2 n^2 \beta_2 L/T_{\mathrm{M}}^2$ acquired by each harmonic in the propagation in the dispersive delay line, where $\beta_2$ is the group velocity dispersion $\beta_2 = \mathrm{d}^2\beta/\mathrm{d}\omega^2$, $\beta$ is the propagation constant, $\omega$ is the optical angular frequency, and $L$ is the line length. Consequently, Eq. (1) can be rewritten in the form:

$$E_{\mathrm{mult}}(t) = \sum_{n=-\infty}^{\infty} c_n \exp(-\mathrm{i}2\pi^2 n^2 \beta_2 L/T_{\mathrm{M}}^2) \times \exp(\mathrm{i}2\pi nt/T_{\mathrm{M}}). \qquad (2)$$

One can see from Eq. (2) that the multiplied pulses will be compressed if

$$\exp\left(-\mathrm{i}2\pi^2 n^2 \beta_2 L/T_{\mathrm{M}}^2\right) = 1$$

and this yields the condition

$$\pi|\beta_2|L/T_{\mathrm{M}}^2 = m \quad (m = 1, 2, \ldots). \qquad (3)$$

If condition (3) is met, the superposition of the phase modulated multiplied pulses gives the following pulses:

$$E_{\mathrm{mult}}(t) = \sum_{n=-\infty}^{\infty} c_n \exp(\mathrm{i}2\pi nt/T_{\mathrm{M}}).$$

These pulses are identical to those, which would be obtained in the conventional compression method at the end of the dispersive delay line with dispersion $\beta_2 L$ defined from condition (3). Therefore, we shall call this dispersive delay line as an equivalent line.

The results obtained are illustrated in Fig. 2. In the conventional compression method, the phase

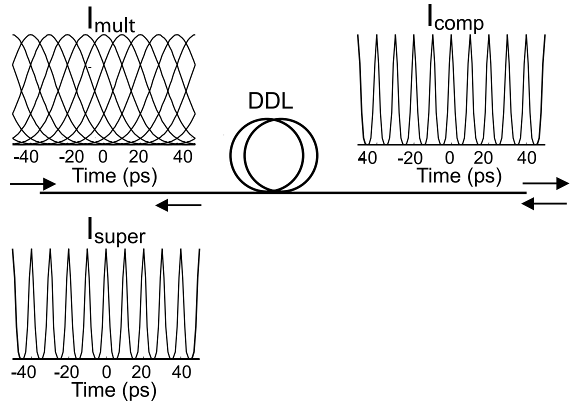


Fig. 2. Illustration for explanation of the proposed method: superposition of the phase modulated multiplied pulses $I_{\mathrm{mult}}$ in our method yields in condition (3) the same multiplied compressed pulses $I_{\mathrm{super}}$ as the pulses $I_{\mathrm{comp}}$ at the output of an equivalent dispersive delay line (DDL) in the conventional compression method.

modulated multiplied pulses $I_{\mathrm{mult}}$ propagate through the equivalent dispersive delay line (DDL) with dispersion $\beta_2 L$ and transform to the compressed pulses $I_{\mathrm{comp}}$. Eq. (2) can be considered as representing propagation of these compressed pulses backwards through the same line but with opposite dispersion $-\beta_2 L$. It is clear that in this case, the dispersion is compensated and the output pulses $I_{\mathrm{super}}$ are identical to the input pulses $I_{\mathrm{mult}}$. In other words, the pulses $I_{\mathrm{super}}$ are the result of the superposition of the multiplied pulses $I_{\mathrm{mult}}$. On the other hand, condition (3) represents the integer temporal Talbot effect [2]. According to this effect, in condition (3), the pulses at the input and output of such a dispersive delay line are identical (temporal self-imaging [2–4]). This implies that the superposition of the phase modulated multiplied pulses $I_{\mathrm{mult}}$ results in the same multiplied compressed pulses $I_{\mathrm{comp}}$ as at the end of the equivalent dispersive delay line in the conventional compression method. (We assume that there is no overlap between the multiplied compressed pulses.)

Condition (3) can be written in a more general form

$$\pi|\beta_2|L/T_{\mathrm{M}}^2 = m/p, \tag{4}$$

where $m$ and $p$ are integers. If $m$ and $p$ have no common factor, condition (4) describes the fractional temporal Talbot effect [5]. In this case, the field amplitude of the pulse train at the dispersive delay line output can be expressed, as in [6], in the form:

$$E_{\mathrm{out}}(t) = \sum_{n=0}^{p-1} C(n,m,p) E_{\mathrm{in}}(t - nT/p), \tag{5}$$

where $E_{\mathrm{in}}(t)$ is the input field amplitude of the pulses, and the coefficients $C(n,m,p)$ are given by

$$C(n,m,p) = (1/p) \sum_{q=0}^{p-1} \exp[(2\mathrm{i}\pi q/p)(n - mq)]. \tag{6}$$

For the case $m = 1$, $p = 2$, it can be obtained, according to (5) and (6), that $C(0,1,2) = 0$, $C(1,1,2) = 1$ and $E_{\mathrm{out}}(t) = E_{\mathrm{in}}(t - T/2)$. It means that in this case, the pulses at the input and output of a dispersive delay line are identical, but shifted by half a period. In the general case of the fractional Talbot effect, the pulses at the output of a dispersive delay line, as it follows from (5) and (6), have the same intensity shape as the input pulses but the output pulse rate is multiplied by $p$, if $p$ is odd or by $p/2$, if $p$ is even. This implies that superposition of the multiplied pulses gives in this case additional multiplication of the original pulses.

Note that our compression method as well as the temporal Talbot effect can be exactly realized only for infinite pulse trains. Nevertheless, the temporal Talbot effect is observed approximately for a finite number of pulses [7]. This occurs under the condition that the total number of the pulses is much larger than the number of the pulses interacting with the central pulse in the dispersed pulse burst. Accordingly, the pulse compression in our method will be still performed if the pulse trains are finite or the periodic pulses have timing or power jitter. The time interval, in which the pulses remain approximately constant, should be in the latter case much larger than the time of the multiplied pulse interaction. For instance, this interaction time for the pulses shown in Figs. 1 and 2 is 170 ps.

Calculation of the pulse propagation can be accomplished using the temporal analog to the ABCD law for spatial Gaussian beams [8]. The dispersion $\beta_2 L$ of the equivalent dispersive delay line can be found (with the assumption of zero chirp in the pulse after the compression) through the given pulse durations $\tau_0$ and $\tau_{\mathrm{c}}$, before and after the compression, respectively (where $\tau$ is the $1/e$ pulsewidth):

$$|\beta_2|L = (\tau_{\mathrm{c}}/2)^2[(\tau_0/\tau_{\mathrm{c}})^2 - 1]^{1/2}. \tag{7}$$

It must be noted that although in reality there is no dispersive delay line in our method, the phase modulation of the pulses, $\varphi = Ct^2/2$ (where $C$ is frequency chirp), is calculated for dispersion (7) of the equivalent dispersive delay line

$$1/C = \beta_2 L[1 + (\tau_{\mathrm{c}}^2/4\beta_2 L)^2].$$

Substitution of $\beta_2 L$ from (7) into condition (4) gives the relation for the pulse compression ratio $r = \tau_0/\tau_{\mathrm{c}}$:

$$(r^2 - 1)^{1/2} = 4mr^2T^2/(\pi p \tau_0^2 M^2).$$

For example, the following parameters were chosen for the calculation of the pulses in Figs. 1 and

2: $\tau_0 = 35$ ps, $M = 10$, $T = 100$ ps, $m = 1$, $p = 1$. In this case, after the appropriate phase modulation and multiplication of the pulse repetition rate from 10 to 100 GHz, the pulse is compressed from 35 to 3.6 ps. It is interesting to note that the pulse compression can occur for both, positive and negative signs of the pulse chirp.

The approach using an equivalent dispersive delay line is very useful because it allows including into consideration not only cases of the fractional Talbot effect but also pulses of an arbitrary form. Indeed, condition (4) is independent of the pulse shape. We shall use this for explanation of the experimental results.

In the experimental demonstration of the proposed method, we realized a particular case of light compression: generating the optical pulses from cw radiation [9,10]. In the conventional method, laser radiation is sinusoidally phase modulated and then propagated through a dispersive delay line. In our method, the multiplication substitutes for propagation in the dispersive delay line. However, the dispersive delay line can be considered as equivalent, similar to that shown in Fig. 2. In the equivalent dispersive delay line, a sinusoidal phase modulation acts like a number of "time lenses" [11], where each one "focuses" the pulse from cw radiation. According to the proposed method, the sinusoidal phase modulation and multiplication yield the same pulses as those "focused" in the equivalent dispersive delay line in the conventional method, but multiplied by $M$ (or additionally multiplied), if condition (4) is met and if there is no overlap between the multiplied compressed pulses. The dispersion $\beta_2 L$ for which the pulse is optimally "focused" is dependent on the pulse period $T$ and on the modulation index $A$ (the phase modulation amplitude). On the other hand, the dispersion $\beta_2 L$ of the equivalent dispersive delay line can be found from condition (4). From this it is clear that for a chosen period $T_M$ and fixed $p$ we have several values of a modulation index $A$ that gives minimal width pulses after phase modulation and rate multiplication. These values correspond to different values of $m$ in (4).

We have performed numerical simulation of the pulses obtained from cw radiation of wavelength 1543.35 nm after phase modulation of rate $f = 6.25$ GHz and rate multiplication $M = 4$. For the selected rate there are several values for the modulation index that give good quality pulses after pulse multiplication at $M = 4$: $A = 2.2$, 2.405, 3.7, 7.61 rad, where in the second and fourth cases the pulse rate obtained is $6.25 \times 4 \times 2$ GHz. The analysis shows that the cases where $A = 2.2$, 3.7 rad correspond to $p = 4$, $m = 4$ and $m = 2$, respectively. As was shown previously, for $m/p = 1/2$ in (4) the input and output pulses in the dispersive delay line are also identical but shifted by half a period. This shift can be seen in Fig. 3, where the dotted curve shows the pulses obtained by the proposed method after phase modulation with modulation index $A = 3.7$ rad and rate multiplication by $M = 4$, the solid and dashed curves show, respectively, for comparison the intensity and phase of the pulses that would have been "focused" in the equivalent dispersive delay line (without multiplication) for $m/p = 1/2$. The dispersion of this equivalent line is $DL = -201.5$ ps/nm (where $D = -2\pi c\beta_2/\lambda^2$, $c$ is the velocity of light and $\lambda$ is the wavelength). The cases where $A = 2.405$, 7.61 rad correspond to $p = 4$, $m = 3$ and $m = 1$, respectively. In this case, the shape of the pulse's intensity at the dispersive line output is also reproduced, however with a doubled rate. This additional pulse multiplication can be

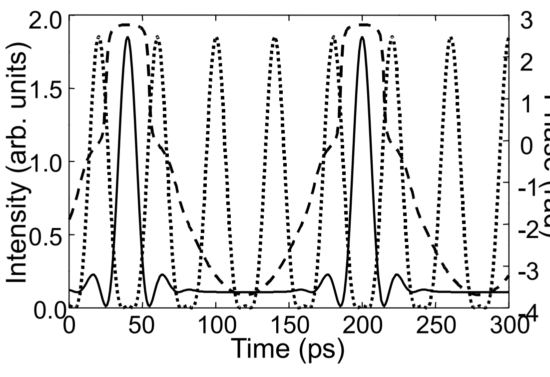


Fig. 3. Numerical simulation: the pulses obtained from cw radiation after phase modulation of frequency $f = 6.25$ GHz, modulation index $A = 3.7$ rad and after rate multiplication of $M = 4$ (dotted curve); the intensity (solid curve) and phase (dashed curve) of the pulses after the same phase modulation and propagation (without multiplication) through an equivalent dispersive delay line with dispersion of $DL = -201.5$ ps/nm, which corresponds to $m/p = 1/2$ in (4).

seen in Fig. 4, which shows calculation of the pulses obtained by the proposed method (dotted line) and at the output (solid line) of the equivalent dispersive delay line (without multiplication) for $A = 7.61$ rad, $M = 4$, $p = 4$, $m = 1$. The dispersion of the equivalent line in this case is −100.75 ps/nm. It must be noted that in Figs. 3 and 4 the pulses after rate multiplication (dotted curves) do not represent an exact 4 and 8 time repetition of the pulses, "focused" in the equivalent delay line (solid curves). This is due to the fact that the intensity and phase between the "focused" pulses are not equal to zero and the pulse superposition at the rate multiplication is of a more complex nature. This also explains the slight difference from the optimal "focusing" for the found values of $A$ (if we consider that the "focusing" is optimal when the phase during the pulse is constant). For instance, the optimal "focusing" for $DL = -201.5$ ps/nm is achieved for $A = 3.9$ rad instead of 3.7 rad.

In the experiment, we accomplished rate multiplication of optical pulses with fiber Bragg gratings (Fig. 1(a)). In [12], a sampled Bragg grating was used for this purpose. We proceed from the assumption that the complex spectra of the signal and the same signal with repetition rate multiplication of $M$ are different by the factor $F(\omega)$ [13]:

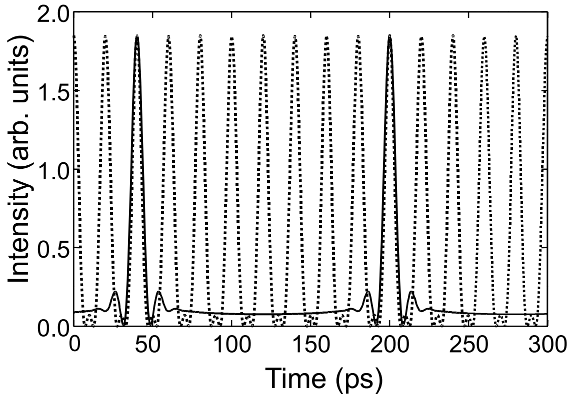


Fig. 4. Numerical simulation: the pulses obtained from cw radiation after phase modulation of frequency $f = 6.25$ GHz, modulation index $A = 7.61$ rad and after rate multiplication of $M = 4$ (dotted curve); the pulses after the same phase modulation and propagation (without multiplication) through a equivalent delay line with dispersion of $DL = -100.75$ ps/nm (solid curve), which corresponds to $p = 4$, $m = 1$ in (4).

$$F(\omega) = \exp[-i\omega(1 - 1/M)T/2] \times [\sin(\omega T/2)/\sin(\omega T/2M)]. \quad (8)$$

In other words, the spectral dependency of the overall reflectivity of the gratings should be of the form (8). The results of the calculation of $|F(\lambda)|^2$ according to (8) for $f = 6.25$ GHz, $M = 4$ are presented in Fig. 5 (dotted curve). It is known that for low reflectivity the spatial envelope of the Bragg gratings should be the Fourier transform of the reflection spectrum. The Fourier transform of (8) results in $M\,\delta$ functions. In other words, to obtain reflectivity in the form of (8), $M$ point Bragg gratings must be written in the fiber, where the distance between them provides the time delay $T/M$. It is clear that in reality the lengths of the gratings should be small compared to the distance between them, so that the maxima in the reflection spectrum differ little from one another.

The Bragg gratings were written in a boron doped photosensitive fiber by cw UV radiation ($\lambda = 244$ nm) using a phase mask. The reflectivity of each grating was 2%. The positioning accuracy of our system of 1 μm was enough to provide the necessary time delay of the pulses between the Bragg gratings, but was not sufficient to obtain reflection from all the gratings in the same phase. To solve this problem, after the gratings were written, the intervals between the gratings were irradiated with UV light without the mask. In this way, the optical path between the gratings was changed until the desired spectral shape of reflec-

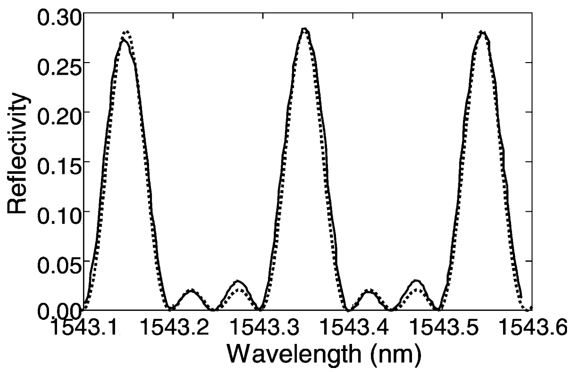


Fig. 5. The measured reflectivity (solid curve) of four Bragg gratings. The normalized function $|F(\lambda)|^2$ (dotted curve) is calculated according to (8) for $M = 4$ and $f = 6.25$ GHz.

tivity was achieved [14]. Fig. 5 (solid curve) shows the reflection spectrum measured with the help of a tunable laser diode for the four fiber Bragg gratings written. The length of each grating is 0.3 mm, the distance between the centers of each two gratings is 4 mm.

In the experiment, the pulses were created from cw radiation of a tunable laser diode. The light, sinusoidally phase modulated by a $LiNbO_3$ electrooptic modulator, was reflected from the fiber Bragg gratings and with a circulator was directed to a photodetector then an oscilloscope (both with 50 GHz bandwidth). To obtain the desired pulses, fine tuning was required of the laser wavelength, the modulation frequency and the modulation index. According to our simulations this tuning should be performed with the following resolutions: $\Delta\lambda = 0.005$ nm, $\Delta f = 0.01$ GHz, and $\Delta A = 0.1$ rad. Fig. 6 shows the pulses obtained with rate multiplication $M = 4$ using four fiber Bragg gratings. The phase modulation frequency and modulation index were 6.25 GHz and 3.3 rad, respectively. The background between the adjacent pulses in Fig. 6 is caused, most likely, by insufficient time resolution of the detector and oscilloscope.

We also obtained pulses from cw radiation by rate multiplication of $M = 2$. The multiplication was performed by two Bragg gratings spaced 8 mm apart. The experimental results are presented in Fig. 7 (solid curve), where the phase modulation frequency was $f = 6.414$ GHz and modulation index was $A = 1.7$ rad. For comparison, the dotted curve in Fig. 7 shows the simulation results for the same modulation frequency and $A = 1.6$ rad. The presence of small additional peaks on the experimental curve is explained by the difference between the experimental value of the modulation index and that used in simulation. Calculation shows that for this case, the almost optimal "focusing" by the equivalent dispersive delay line is obtained for dispersion $|D|L = 765$ ps/nm which corresponds to $m/p = 1/2$ in (4).

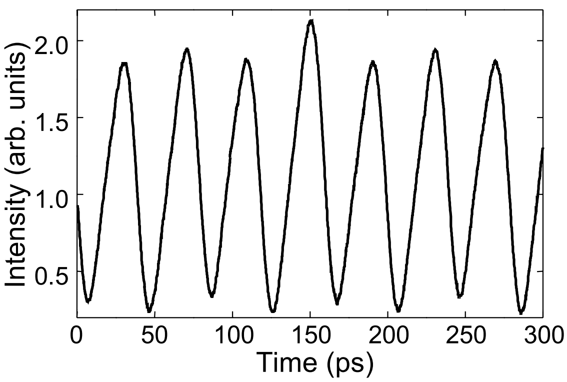


Fig. 6. The pulses obtained experimentally from cw radiation of a laser diode after sinusoidal phase modulation (modulation frequency $f = 6.25$ GHz and modulation index $A = 3.3$ rad) and after rate multiplication of $M = 4$.

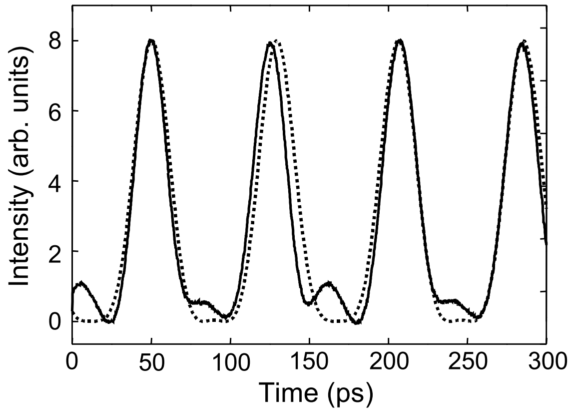


Fig. 7. The pulses obtained from cw radiation after sinusoidal phase modulation with frequency of 6.414 GHz and rate multiplication of $M = 2$: experiment with $A = 1.7$ rad (solid curve) and calculation for $A = 1.6$ rad (dotted curve).

In conclusion, we proposed a novel method for compression of periodic optical pulses based on repetition rate multiplication that does not require propagation of the pulses in a dispersive delay line. This method was explained using the temporal Talbot effect. We also presented experimental demonstration of the method, where optical pulses were generated from cw radiation using sinusoidal phase modulation and rate multiplication. The proposed method can have wide applications because it simultaneously implements two important requirements of many fields, for example, of optical communications: pulse compression and pulse repetition rate multiplication. In addition, the method proposed enables the construction of more compact and thus more stable devices for compression or generation of pulses. For instance, for compression of the chirped optical pulses of a laser diode, hundreds of meters up to several kilometers of fiber are used. For comparison, the length of the repetition rate multiplier in our experiments was 14 mm (for $M = 4$). A repetition rate multiplier can also be constructed on the same chip as a semiconductor laser, which would further increase the compactness of the device.

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