Experimental demonstration of localization in the frequency domain of mode-locked lasers with dispersion

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Mode-locked lasers with intracavity dispersion are experimentally shown to exhibit localization behavior in their frequency domain. The localization, with its typical exponential spectrum structure, is analogous to that which occurs for the quantum kicked rotor. The experimental demonstration of our optical kicked rotor is done with a long mode-locked dispersive fiber laser. The localization effect sets a basic limit on the spectrum bandwidth and the minimum pulse width in such lasers. It also provides a special experimental test bed for the study of optical kicked rotors and localization effects.

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A recent work[1] showed theoretically the occurrence of localization in the frequency domain of light pulses that propagate in a dispersive optical fiber with repeated modulation kicks at equally spaced locations along the fiber. An excellent realization of such an optical kicked system is the active mode-locked laser with strong intracavity dispersion. Localization means that the steady-state development of sidebands is limited, depending on the modulation parameters and the intracavity dispersion, resulting in a limit on the pulse widths of such lasers, as well as a typical exponential signature of the spectrum envelope. This kind of localization is similar to that which occurs in the phase space of the quantum kicked rotor,[2–4] which is related to Anderson localization for electrons in one-dimensional disordered solids.[5–7] The localization is expressed by the rapid exponential decay of the wave function. Finding experimental evidence for localization in quantum kicked rotors has been a constant challenge, and very few direct observations have been reported to date. The first one was performed using laser-cooled atoms.[8–10] More recently, localization in a classical optical system was demonstrated experimentally for free-space light propagation, kicked by a series of diffraction gratings.[11,12]

In this Letter we present an experimental demonstration of the predicted localization in mode-locked lasers with intracavity dispersion. The study gives new insight into the thoroughly investigated mode-locked laser system and also opens new possibilities for studying localization. The mode-locked laser is an almost ideal system for such studies, in which a single entity can be measured, compared with, for example, the accessibility to only the macroscopic properties of electrons in solids. Our experiment is performed with long fiber lasers with normal dispersion to eliminate the possibility of soliton formation (which was also restricted by power control), which also gives exponential spectral behavior. Nevertheless, we performed experiments with regular fibers that had anomalous dispersion at the 1550-nm wavelength range and obtained similar results. The localization is manifested by the spectrum confinement and its exponential signature, compared with the regular broad and Gaussian spectrum. In the present experiment the measurements taken are of the long-term steady-state behavior of the spectrum and not the evolution that follows the spectrum dependence on the kick number. One can view the mode locking of a laser as successive modulation of a circulating light pulse in a dispersive cavity, such as a long fiber span. In this case, propagation of the pulse electric field envelope, ψ, can be simply described by a Schrödinger-like equation[1,13]:

$$i\,\frac{\partial \psi}{\partial z} = \frac{\beta_2}{2}\,\frac{\partial^2 \psi}{\partial T^2} + \kappa\,\cos(\Omega T)\sum_N \delta(z - N z_0)\psi\,, \qquad (1)$$

where the propagation is along z in the cavity, $T = t - z/v_g = t - \beta_1 z$ is the internal pulse time variable (relative to the center of the pulse), $v_g = 1/\beta_1$ is the group velocity, and β_2 is the group-velocity disperion (responsible for pulse broadening), which serves here as the Planck constant in the quantum case. The potential results from perturbation by the time-dependent phase modulation with a frequency Ω and an amplitude κ (which is real for phase modulation or FM mode locking, and imaginary for amplitude modulation or AM mode locking). The distance between successive kicks is the cavity round-trip length, $z_0 = l$. The phases of the modulations (kicks) are automatically synchronized for mode locking, where the modulation frequency equals one of the harmonics of the cavity resonance.[1] We did not take into account other components in the cavity, such as sections with gain and absorption, which are assumed to provide the oscillation condition. These components are assumed to have a relatively broad spectral response that does not affect the spectral behavior in the localized regime. Considering the evolution of the pulse in the frequency domain on propagation in the fiber ring laser, we realize that it undergoes successive modulations (kicks), and therefore the number of the sidebands (harmonics) increases. The propagation between the kicks adds extra phases, $\frac{1}{2}\gamma n^2$, where $\gamma = \beta_2 z_0 \Omega^2$, that depend on the intracavity dispersion and the sideband order n.[1] In a mode-locked laser,

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where the modulation is applied repeatedly, the naive expectation is that the number of sidebands, and thus the overall frequency bandwidth, increases in a diffusive manner until limited by components in the system. However, the wave behavior with the dispersive propagation between the successive kicks produces the localization effect.

The theoretical study predicts that for γ/π rational the spectrum envelope is extended, whereas if it is irrational it is exponentially localized.[1,2] The case in which $\gamma/2 = m\pi$, for integer m, is related to special resonances in our optical system, as was recently demonstrated,[14] showing the dispersion-mode laser operation with a broad Gaussian spectrum. In the localized regime, which is the focus of this Letter, the envelope of the wave spectrum behaves as $\exp(-|n - n_0|/\xi)$, where ξ is the localization length. The analysis shows that the localization width is $\xi \sim \kappa^2/4$ harmonic orders, in specific parameter regimes.[2] The transition from diffusion to localization usually occurs after a few kicks, given by $\sim\kappa^2/8$. The present experiment, however, shows the long-term steady-state behavior of the spectrum.

The experimental system, shown in Fig. 1, consists of a long fiber ring laser, of the order of a few kilometers, with significant intracavity dispersion. We used a high positive group-velocity dispersion fiber with a length of $l = 1$ km, and $\beta_2 \approx +142$ ps^2/km (at wavelengths near 1550 nm) to eliminate soliton formation in the laser. We also examined lasers consisting of fiber with anomalous dispersion of $\beta_2 \approx -20$ ps^2/km and a length of $l = 50$ km. (β_2 varies slightly when there are small changes of the lasing wavelength in the experiment as a result of different operating conditions.) The laser cavity contained an erbium-doped fiber amplifier pumped by a 980-nm wavelength diode laser and a $LiNbO_3$ modulator for active mode locking. In the experiments presented here we used amplitude modulation but also studied the application of phase modulation. Numerical studies of the propagation equation show some differences between the two modulation cases, but both give localization effects. Practically, the lack of a strong loss mechanism in the laser operation for the phase modulation case can alter the steady-state operation of the laser. The modulation frequency that provided the mode locking was a few gigahertz and matched one of the high-order (approximately 10^5–10^6) harmonics of the basic cavity resonance.

The experimental results showing the laser output are given in Figs. 2–4. One can see the confined spectra with the typical exponential envelope, as predicted,[1] for localization in the temporal frequency domain. As mentioned above, the spectrum depends mainly on the modulation depth, κ, and in some regimes on γ and, therefore, on the modulation frequency, Ω, and the fiber length. In our experiments the confinement width was a few sidebands (harmonic orders), corresponding to $\kappa \sim 1$–2.5. In Fig. 2 the comparison of the experimental results can be seen between the localized spectrum with an exponential fit and the spread Gaussian spectrum at resonance for one of the dispersion modes,[14] with a Gaussian fit, as discussed above. The first resonance ($m = 1$), $\gamma = \pi$, in this laser system with 50-km fiber with anomalous dispersion occurs at $\Omega/2\pi = 12.8$ GHz. The fitted curves in Fig. 2 match the experimental data nicely. In Fig. 3, output spectra for 1 km of fiber laser and

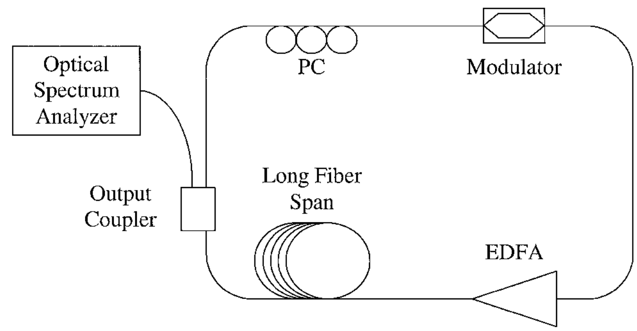

Fig. 1. Experimental configuration of the mode-locked laser, consisting of a fiber ring cavity with an erbium-doped fiber amplifier and a $LiNbO_3$ modulator. PC, polarization controller; EDFA, erbium-doped fiber amplifier.

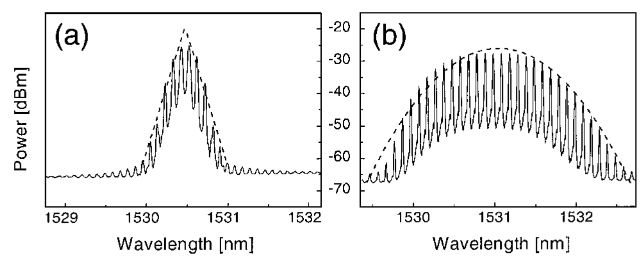

Fig. 2. Output spectra for $l = 50$ km and $\beta_2 \approx -20$ ps^2/km: (a) Corresponding to the localization case. Here $\Omega/2\pi = 13.8$ GHz. The exponential envelope fit is centered around 1530.47 nm, and its width is 0.0535 nm, giving approximately $\xi \approx 0.485$ sidebands. (b) At resonance with the extended spectrum. Here $\Omega/2\pi = 12.8$ GHz. The Gaussian fit is centered around 1531.04 nm, and its width is 0.37 nm, or approximately $\approx$3.61 sidebands.

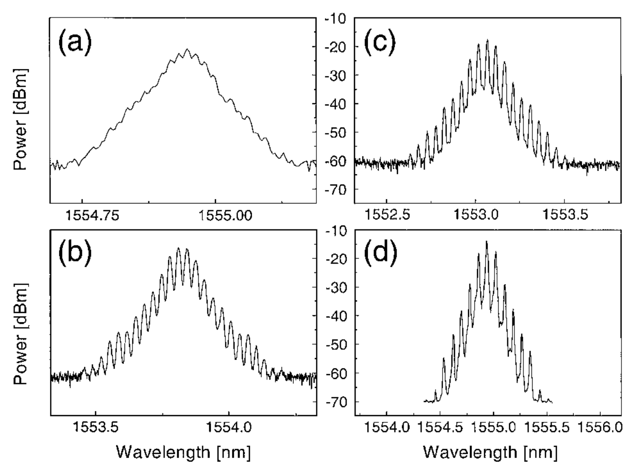

Fig. 3. Output spectra showing localization for $l = 1$ km and $\beta_2 \approx +142$ ps^2/km (at $\lambda = 1550$ nm) and (a) $\Omega/2\pi = 2$ GHz, (b) $\Omega/2\pi = 4$ GHz, (c) $\Omega/2\pi = 6$ GHz, and (d) $\Omega/2\pi = 10.1$ GHz. Note the difference in the wavelength scale, presented to show the dependence of the spectra on the number of the sidebands.

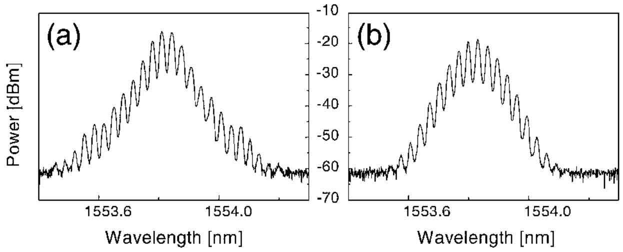

Fig. 4. Output spectra for $l = 1$ km and $\beta_2 \approx +142\ \mathrm{ps}^2/\mathrm{km}$ (at $\lambda = 1550$ nm) and $\Omega/2\pi = 4$ GHz for (a) strong and (b) a weak (approximately 15 times weaker) modulation amplitudes. Note that for weak modulation the spectrum no longer has an exponential shape.

different modulated frequencies are presented. It can be seen that there is a decrease in the confinement width of the output spectra as the modulation frequency is increased, which results from the decrease in κ caused by the degradation of the frequency response of the modulator at higher frequencies. We note the difference in the wavelength scale in Fig. 3, which shows the dependence of the spectra on the number of sidebands. The graphs show that the number of sidebands decreases as the modulation frequency, Ω, increases, and thus κ decreases, as expected from the theory. In Fig. 4, output spectra are presented for the 1-km-long mode-locked laser, where the modulation amplitude is decreased while a specific frequency is applied. Here we observe that for very small κ the spectral envelope of the laser does not have an exponential shape.

The occurrence of localization limits the short-pulse formation in such mode-locked lasers. This spectrum in the localization regime is different from the standard broad spectra of mode-locked lasers. Nevertheless, as was mentioned above, pulse and broad spectrum formation are still possible in mode-locked lasers with significant dispersion at special regions that correspond to resonances of the dispersion modes.[14] The dispersionless mode-locked laser with a broad spectrum is the trivial example of this family. In the future we plan to present a detailed study of such lasers near resonance.

In conclusion, we have experimentally shown that mode-locked lasers with intracavity dispersion exhibit localization effects in their frequency domain, in a way similar to the quantum kicked rotor. The localized laser spectra have a typical exponential spectrum structure, limiting the formation of short pulses in the laser. This kicked laser system is an attractive experimental system that provides a demonstration of localization in general and particularly in optics.

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References

1. B. Fischer, A. Rosen, and S. Fishman, Opt. Lett. **24,** 1463 (1999).
2. S. Fishman, D. R. Grempel, and R. E. Prange, Phys. Rev. Lett. **49,** 509 (1982).
3. D. R. Grempel, R. E. Prange, and S. Fishman, Phys. Rev. A **29,** 1639 (1984).
4. F. Haake, *Quantum Signatures of Chaos* (Springer, New York, 1991).
5. P. W. Anderson, Phys. Rev. **109,** 1492 (1958).
6. For reviews, see D. J. Thouless, in *Critical Phenomena, Random Systems, Gauge Theories*, Proceedings of the Les-Houches Summer School, K. Osterwalder and R. Stora, eds. (North-Holland, Amsterdam, 1986), p. 681.
7. I. M. Lifshits, S. A. Gredeskul, and L. A. Pastur, *Introduction to the Theory of Disordered Systems* (Wiley, New York, 1988).
8. F. L. Moore, J. C. Robinson, C. F. Barucha, B. Sundaram, and M. G. Raizen, Phys. Rev. Lett. **75,** 4598 (1995).
9. B. G. Klappauf, W. H. Oskay, D. A. Steck, and M. G. Raizen, Phys. Rev. Lett. **81,** 1203 (1998).
10. H. Ammann, R. Gray, I. Shvarchuck, and N. Christensen, Phys. Rev. Lett. **80,** 4111 (1998).
11. B. Fischer, A. Rosen, A. Bekker, and S. Fishman, Phys. Rev. E **61,** R4694 (2000).
12. A. Rosen, B. Fischer, A. Bekker, and S. Fishman, J. Opt. Soc. Am. B **17,** 1579 (2000).
13. G. P. Agrawal, *Nonlinear Fiber Optics*, 2nd ed. (Academic, San Diego, Calif., 1995).
14. B. Fischer, B. Vodonos, S. Atkins, and A. Bekker, Opt. Lett. **25,** 728 (2000).