

## PHOTOREFRACTIVE SATURABLE ABSORPTIVE AND DISPERSIVE OPTICAL BISTABILITY

Shimon WEISS and Baruch FISCHER

*Department of Electrical Engineering, Technion, Israel Institute of Technology, Haifa 32000, Israel*

Received 15 November 1988

It is shown that photorefractive wave-mixing gives saturable gain, saturable absorption and Kerr-like nonlinear phase retardation. An analogy between photorefractive two-beam coupling and two-level atom system is then drawn. We investigate optical bistability in a ring cavity, utilizing these phenomena, and compare our results with classical absorptive and dispersive optical bistability.

In classical nonlinear optics, laser beams alter the optical properties of the medium and in turn, the propagation of the beams themselves. Saturable absorption, and Kerr effect are examples of third order nonlinearities, which are the origin of many interesting effects, such as self-focusing and self phase modulation [1]. Adding optical feedback to such nonlinear media leads to other properties such as dispersive and absorptive optical bistability (OB), multi-stability, self pulsation and chaos [2].

The photorefractive (PR) effect, which is also a third order process [3], differs substantially from the conventional nonlinearities in the following points: (i) it is nonlocal, (ii) the effect is dependent on the ratios of the light beams' intensities (it is sensitive to the visibility of the interference pattern, but not to the total light intensity which affects only the time response), (iii) the material must be noncentrosymmetric in order to have nonzero electrooptic coefficients. The simplest PR configuration, the two-beam coupling (2-BC) (shown in fig. 1), has been extensively studied in the past [4], where it was mostly considered and used for its light gain capabilities.

In this work we point out and elaborate on other important aspects of this process. We show that 2-BC displays saturable absorption and Kerr-like nonlinear dispersion, which give absorptive-like and dispersive-like bistabilities, when a feedback is added. In fact, under the proper conditions, discussed be-

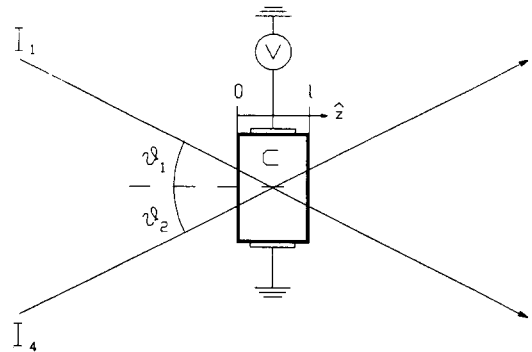


Fig. 1. Two-beam coupling scheme. C is the photorefractive crystal, V a voltage source, and  $\theta_1$  and  $\theta_2$  are the angles of the two beams,  $I_1$  and  $I_4$ , with respect to the normal to the crystals' face.

low, the PR 2-BC system can simulate the conventional nonlinear local effects.

Fig. 1 describes the basic configuration for 2-BC with transmission grating. We consider here nondegenerate mixing, and the possible application of a DC electric field on the PR crystal. Our treatment follows the guidelines, definitions and notations of ref. [5]. The two waves, denoted by 1 and 4, interact in the crystal via the photoinduced grating. The interaction is described by the following coupled wave equations

$$dA_1/dz = -(\gamma/I_0) A_1 |A_4|^2 - \alpha A_1, \quad (1a)$$

$$dA_4^*/dz = (\gamma/I_0) A_4^* |A_1|^2 - \alpha A_4^*, \quad (1b)$$

where  $A_j$  is the complex amplitude of the  $j$ th beam ( $j=1, 4$ ),  $I_j$  is its intensity ( $I_j=|A_j|^2$ ),  $I_0(z)=I_1(z)+I_4(z)$  is the total light intensity at  $z$ ,  $\alpha$  is the normalized linear absorption coefficient and  $\gamma$  is the complex coupling constant, as defined in ref. [5]. (Note that these definitions are valid for  $\theta_1=\theta_2$  in fig. 1.) The physics of the PR effect lies in  $\gamma$  and was first successfully modeled, via band transport mechanism, by Kukhtarev et al. [6]. With  $A_j(z)=\sqrt{I_j(z)} \exp[i\psi_j(z)]$ ,  $\psi_j(z)=k_j z + \varphi_j(z)$ ,  $\Gamma=2\text{Re}(\gamma)$ ,  $\Gamma'=\text{Im}(\gamma)$ ,  $k_j$  being the  $j$ th beam wave vector and  $\varphi_j(z)$  the additional phase due to the nonlinear process, we obtain from eq. (1),

$$\frac{dI_1}{dz} = -\Gamma \frac{I_1 I_4}{I_0} - 2\alpha I_1 = -\frac{\Gamma I_1}{1+I_1/I_4} - 2\alpha I_1, \quad (2a)$$

$$\frac{dI_4}{dz} = \Gamma \frac{I_1 I_4}{I_0} - 2\alpha I_4 = \frac{\Gamma I_4}{1+I_4/I_1} - 2\alpha I_4, \quad (2b)$$

$$d\varphi_1/dz = -\Gamma' I_4/I_0, \quad (2c)$$

$$d\varphi_4/dz = -\Gamma' I_1/I_0. \quad (2d)$$

The solutions are

$$I_1(z) = I_1(0) \frac{[1+I_{41}(0)] \exp(-2\alpha z)}{1+I_{41}(0) \exp(\Gamma z)} = I_0(0) \frac{\exp(-2\alpha z)}{1+I_{41}(0) \exp(\Gamma z)}, \quad (3a)$$

$$I_4(z) = I_4(0) \frac{[1+I_{14}(0)] \exp(-2\alpha z)}{1+I_{14}(0) \exp(-\Gamma z)} = I_0(0) \frac{\exp(-2\alpha z)}{1+I_{14}(0) \exp(-\Gamma z)}, \quad (3b)$$

$$\varphi_1(z) = -\frac{\Gamma'}{\Gamma} \ln \left| \frac{1+I_{41}(0) \exp(\Gamma z)}{1+I_{41}(0)} \right|, \quad (3c)$$

$$\varphi_4(z) = \frac{\Gamma'}{\Gamma} \ln \left| \frac{1+I_{14}(0) \exp(-\Gamma z)}{1+I_{14}(0)} \right|. \quad (3d)$$

Here  $I_{41}=I_4/I_1$  and  $I_{14}=I_1/I_4$ . In the special case of  $\Gamma=0$ , eqs. (3c) and (3d) give

$$\varphi_1(z) = -\frac{\Gamma'}{\Gamma} \ln \left| \frac{1+I_{41}(0)(1+\Gamma z)}{1+I_{41}(0)} \right| = -\Gamma' z \frac{I_{41}(0)}{1+I_{41}(0)} = -\Gamma' z \frac{I_4(0)}{I_0(0)}, \quad (4a)$$

$$\varphi_4(z) = \frac{\Gamma'}{\Gamma} \ln \left| \frac{1+I_{14}(0)(1-\Gamma z)}{1+I_{14}(0)} \right| = -\Gamma' z \frac{I_{14}(0)}{1+I_{14}(0)} = -\Gamma' z \frac{I_1(0)}{I_0(0)}. \quad (4b)$$

The complex coupling coefficient,  $\gamma$ , is given by [7]

$$\gamma = \gamma_0 \frac{f(E_0)}{1+i\tau\Omega}, \quad (5)$$

where  $\gamma_0$  is the coupling constant for zero applied (or photovoltaic) electric field and degenerate mixing,  $\Omega \equiv \omega_1 - \omega_4 = 0$ . It depends on the geometry of the two beams with respect to the crystal axes, and on the specific material parameters [5]. In diffusion dominant crystals,  $\gamma_0$  is real (the phase shift of the grating with respect to the interference pattern is  $\pi/2$ ).  $\tau$  is the complex time constant of the grating buildup, and  $f(E_0)$  is the contribution of the applied (photovoltaic) electric field to  $\gamma$ , both given by

$$\tau = \tau_0 \left( \frac{E_d + E_p}{E_d + E_\mu} \right) \left( \frac{E_0 + i(E_d + E_\mu)}{E_0 + i(E_d + E_p)} \right), \quad (6)$$

$$f(E_0) = \left( \frac{E_d + E_p}{E_d} \right) \left( \frac{E_0 + iE_d}{E_0 + i(E_d + E_p)} \right), \quad (7)$$

$E_0$  is the applied electric field parallel to the grating wave vector in the crystal.  $E_\mu = \sigma p_d / (\mu k_g)$ ,  $E_d = k_B \tilde{T} k_g / e$ , and  $E_p = e p_d / (\epsilon k_g)$ , where  $p_d$  is the density of traps in the material,  $\sigma$  is the recombination coefficient of electrons or holes with traps,  $\mu$  is electron or hole mobility,  $k_B$  is Boltzmann's constant,  $\tilde{T}$  is temperature,  $e$  is the electron charge,  $\epsilon$  is the permittivity of the material,  $k_g$  is the grating's wave number, and  $\tau_0$  is the zero field time response of the crystal, which is approximately inversely proportional to the total power density of the beams,  $I_0$  [5]. We note that the phase mismatch due to the non-degenerate wave-mixing is negligible in PR crystals, as indicated in ref. [7].

With a proper choice of  $\Omega$  and  $E_0$ , for a given  $\tau_0(I_0)$ ,  $\gamma$  may become an imaginary value. Simple algebra shows that when

$$\Delta > \frac{2(E_d + E_\mu)}{E_\mu} \left( \frac{E_d}{E_d + E_p} \right)^{1/2}, \quad (8)$$

with  $\Delta \equiv \tau_0 \Omega$ , two values exist for the applied electric field, which makes  $\gamma$  imaginary,

$$E_0 = \{A(E_d + E_p)E_\mu \pm [A^2(E_d + E_p)^2 E_\mu^2 - 4E_d(E_d + E_p)(E_d + E_\mu)^2]^{1/2}\} \times [2(E_d + E_\mu)]^{-1}. \quad (9)$$

The sign of  $E_0$  is determined by the sign of  $\Omega$  (since all the other fields are positive). Practically, the smaller value of  $E_0$  should be chosen (high voltage may damage the crystal). In the case of real  $\gamma$ , (i.e.  $\Gamma=0$ ), the two beams exchange energy, but their phases are not affected, as indicated by (3a–d). With negligible linear absorption, the expressions (2a) and (2b) are very similar to those which describe intensity amplification and deamplification in a homogeneously broadened two (four) level system

$$\frac{dI_0}{dz} = \frac{\tilde{\gamma}I_\nu(z)}{1 + I_0(z)/I_s}, \quad (10a)$$

$$\frac{dI_\nu}{dz} = \frac{-\tilde{\alpha}I_\nu(z)}{1 + I_0(z)/I_s}, \quad (10b)$$

where  $\tilde{\gamma}$  and  $\tilde{\alpha}$  are the amplification and absorption coefficients, respectively. Both  $\tilde{\gamma}$  and  $\tilde{\alpha}$  are saturated due to upward/downward transitions, caused by the presence of the optical field at frequency  $\nu$ ,  $I_\nu$ . In the PR case, one beam is amplified (signal) on the expense of the other (pump). This process saturates when the signal grows well above the pump. Then, the visibility of the fringes decreases, the amplitude of the space charge field decreases and this in turn decreases the amplitude of the grating and the coupling coefficient,  $\gamma$  itself. A difference however should be noticed: the saturation intensity in (10a, b) is fixed, whilst in (2a, b), the other beam ( $I_4$  for  $I_1$  and vice-versa) acts as  $I_s$ , and it is position dependent.

The case of imaginary  $\gamma$ , (i.e.  $\Gamma \neq 0$ ) was studied only for Kerr-like media [8]. In PR 2-BC, similar results are found: there is no energy transfer between the two beams, as can be seen from (3a) and (3c), but the two beams experience phase changes due to the nonlinear coupling, as they propagate in the medium. These are given by (4a) and (4b), and are similar to the Kerr effect. In the later, the index of refraction is given by  $n = n_0 + n_2 \langle I \rangle$ , where  $n_0$  is the unperturbed bulk index of refraction,  $n_2$  is the optical Kerr constant and  $\langle I \rangle$  is the average beam intensity.  $n_2$  can be positive or negative, depending on

the material properties (positive or negative Kerr effect).

The PR analog to  $n_2$  is, from (4a) and (4b),  $(n_2)_{\text{eff}} = \Gamma \lambda / 2\pi$ . This factor is multiplied by the intensity ratio  $I_j(0)/I_0(0)$ , (instead of *intensity* in Kerr effect) and by  $z$ , to give the nonlinear dependent phase change. As  $\gamma$  becomes purely imaginary, the gratings' phase shift becomes  $\theta$  or  $\pi$  radians shifted with respect to the interference pattern, for the two values of  $E_0$ , given in eq. (9). It means that both positive and negative Kerr-like nonlinearities can be achieved.

Now we address ourselves to the application of PR-2BC to Optical Bistability. This subject was considered in several experimental works [9–12], which are based on competition between several coupled oscillators. It was also discussed in a few recent theoretical works. In one of them, four wave mixing configuration in cubic PR crystals is considered [13]. It numerically shows the possibility of intrinsic bistability (O.B. without external feedback). A second paper considers two beam couplings in a linear cavity in the special case of  $\pi/2$  phase shift between the interference pattern and the gratings [14]. Another work [15] utilizes a configuration which was studied above for conventional third order four-wave mixing [16], but the mixer is a PR medium. It also shows multi-valued solutions and possible bistabilities.

Traditionally, O.B. is treated with two types of nonlinear effects: absorption and dispersion (or combination of both) and with two prototype schemes: the unidirectional ring and the linear (Fabry-Perot) cavity, drawn in figs. 2(a) and 2(b) respectively. It was shown, that the Maxwell-Bloch equations, describing the interaction of the field with intracavity-level atom system, is simpler to solve for the ring cavity [2]. It eliminates the difficulties arising from the presence of standing waves in the linear cavity. Figs. 3(a) and 3(b) are the two PR-analogs, with an added "idler" (beam  $I_1$ ) to the "active" beam ( $I_4$ ). As will be shown below, the intensity ratio of these two beams will play an important role in PR-OB. We also allow for the application of an electric field on the crystal, as discussed above.

We analyze here only the ring cavity (fig. 3a), where the induced grating is a transmission one, but our treatment may be applied (with some modifi-

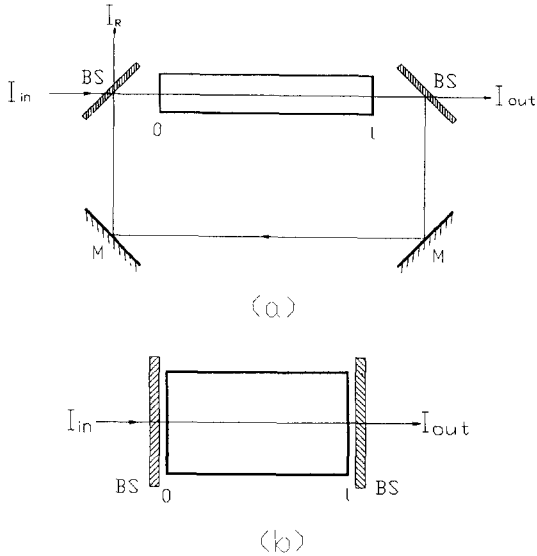


Fig. 2. (a) Bistable unidirectional ring. BS is a beam splitter, and M is a mirror. (b) Bistable linear (Fabry-Perot) cavity.

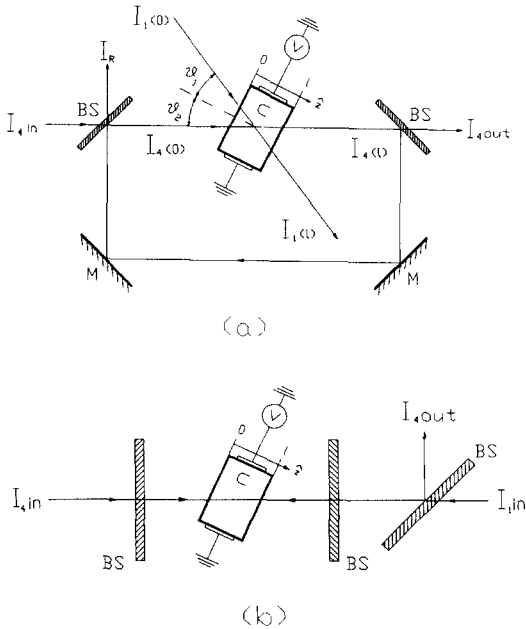


Fig. 3. (a) Photorefractive bistable unidirectional ring. (b) Photorefractive bistable linear cavity.

cations) to the linear cavity (fig. 3b). The later case involves reflection grating, with slightly different coupled wave equations [17], and different boundary conditions.

Our O.B. analysis follows closely the guidelines and notations of Bonifacio and Lugiato [2,18,19] who treated O.B. in conventional nonlinearities. It enables a convenient way of comparing our results with those of refs. [2,18,19]. The boundary condition for the ring cavity is

$$A_4(0) = \sqrt{T} A_{4(in)} + R \exp(+i\delta_0) A_4(l), \quad (11)$$

where  $T$  and  $R$  are the transmissivity and reflectivity of the input and output beamsplitters and

$$\delta_0 \equiv (\omega_4 - \omega_c) / (C/L), \quad (12)$$

$\omega_c = n2\pi c/L$  is the "cavity frequency", nearest to the frequency of the incident field,  $\omega_4$ .  $L$  is the total length of the cavity (including  $l$ ). As discussed above,  $\Delta$  depends on  $I_0$  (the total light intensity), through  $\tau_0$ . Thus,  $\gamma$  is also dependent on  $I_0$ , which is varying with the input intensity. In the following derivations, however, we use a constant  $\Delta$  corresponding to an average total light intensity in the crystal, for the relevant range of input intensities. This considerably simplifies the complicated calculations. We define the amplitude ratio,  $F(z) \equiv A_4^*(z)/A_1(z)$ , and obtain from eq. (1) the following equation for it

$$dF/dz = \gamma F, \quad (13)$$

of which solution is

$$F(z) = F(0) \exp(\gamma z). \quad (14)$$

We also define normalized input and output amplitudes  $x$  and  $y$ ,

$$y \equiv A_{4in} / \sqrt{TA_1(0)},$$

$$x \equiv F(l) = A_{4out}^* / \sqrt{TA_1(l)}, \quad (14)$$

(assuming that  $A_{4in} = A_{4in}^*$ ) and normalized intensities  $Y = y^2$  and  $X = |x|^2$ . Then we find from eqs. (3a), (3c), (14) and (15),

$$A_1(l)/A_1(0) = \eta(X) \exp(-\alpha l)$$

$$\times \exp\{i(\Gamma'/\Gamma) \ln[\eta^2(X)]\}, \quad (16)$$

with

$$\eta(X) \equiv \left( \frac{1 + X \exp(-\Gamma l)}{1 + X} \right)^{1/2}. \quad (17)$$

Now, we rewrite eq. (11),

$$F(0) = Ty + R \exp(-i\delta_0) [A_1(l)/A_1(0)]x, \quad (18)$$

solve for  $Y$  as function of  $X$  and obtain our main result,

$$Y = (X/T^2) \{ [\exp(-\Gamma l/2) - R\eta(X) \exp(-\alpha l)]^2 + 4R\eta(X) \exp[-(\Gamma/2 + \alpha)l] \times \sin^2[\frac{1}{2}[(\Gamma'/\Gamma) \ln(\exp(\Gamma l)\eta^2(X)) - \delta_0]] \}, \quad (19)$$

which is the Optical Bistability state equation for the PR ring cavity. It relates the normalized output intensity  $X$  to the normalized input intensity  $Y$ .

As in the case of "classical" O.B., some more insight may be gained by considering the "mean field limit" [2], in which the intracavity field becomes almost uniform in the cavity. In our case, it is obtained when

$$\alpha = 0, \quad \gamma l \rightarrow 0, \quad T \rightarrow 0, \quad \delta_0 \rightarrow 0, \quad (20)$$

with  $C$ , the bistability parameter [2], redefined for the PR case as:  $C \equiv \gamma_0 l / 2T = \text{const.}$  (instead of  $C \equiv \alpha l / 2T$ ) and  $\theta \equiv \delta_0 / T = (\omega_4 - \omega_c) / (CT / \mathcal{L}) = \text{const.}$  In this limit, for the special case where  $E_0 = 0$  (i.e.  $\Gamma = 2\gamma_0 / (1 + \Delta^2)$ ,  $\Gamma' = -\gamma_0 \Delta / (1 + \Delta^2)$ ), eq. (19) is approximated by

$$Y = X \left[ \left( 1 - \frac{2C}{(1 + \Delta^2)(1 + X)} \right)^2 + \left( \theta + \frac{2C\Delta}{(1 + \Delta^2)(1 + X)} \right)^2 \right]. \quad (21)$$

The result of eq. (21) is almost identical to eq. (31) of ref. [2], where absorptive and dispersive O.B. in homogeneously broadened two-level atom system is considered.

Fig. 4 shows the exact solution for the normalized output intensity,  $X$ , as a function of the normalized input intensity,  $Y$ , according to eq. (19), for  $T = 0.01$ ,  $\alpha = 0$ ,  $\gamma_0 l = -0.3$ ,  $\Delta = 1$  and  $\delta_0 = 0.01$ . The dash-dotted line is the mean field limit (21).

Next we consider two special cases:

(i) *Absorptive-like photorefractive (PR) optical-bistability (O.B.)*

When  $E_0 = \Delta = 0$  and  $\omega_4 = \omega_1 = \omega_c$ , all the amplitudes become real. Eq. (19) then gives

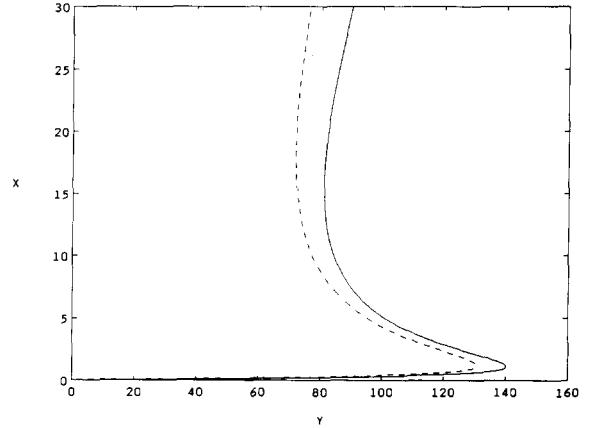


Fig. 4. Normalized output intensity ( $X$ ) versus normalized input intensity ( $Y$ ) for the PR bistable ring cavity. Here  $T = 0.01$ ,  $\alpha = 0$ ,  $\gamma_0 l = -0.3$ ,  $\Delta = 1$  and  $\delta_0 = 0.01$ . The dash-dotted line is the mean field limit (21).

$$Y = (X/T^2) \{ [\exp(-\Gamma l/2) - R\eta(X) \exp(-\alpha l)]^2 + 4R\eta(X) \exp[-(\Gamma/2 + \alpha)l] \sin^2(\delta_0/2) \}. \quad (22)$$

In the mean field limit,

$$Y = X[1 - 2C/(1 + X)]^2, \quad (23)$$

or

$$y = x - 2cx/(1 + x^2). \quad (24)$$

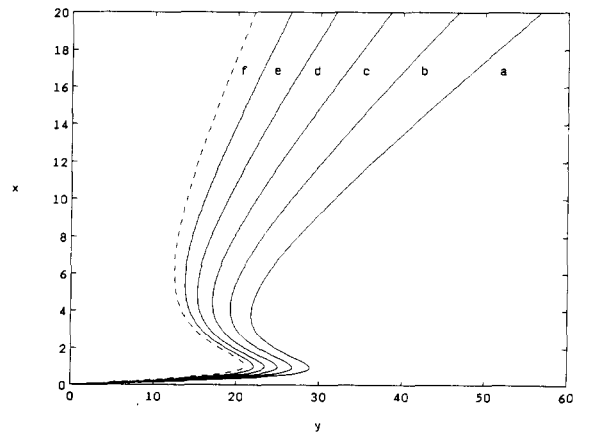


Fig. 5. Normalized output amplitude ( $x$ ) versus normalized input amplitude ( $y$ ). Lines (a) to (e) are the exact solutions, with  $C = -20$ , and  $(\gamma_0 l, T) = (-1, 0.025)$ ,  $(-0.8, 0.02)$ ,  $(-0.6, 0.015)$ ,  $(-0.4, 0.01)$ ,  $(-0.2, 0.005)$  respectively. Line (f) is the mean field limit.

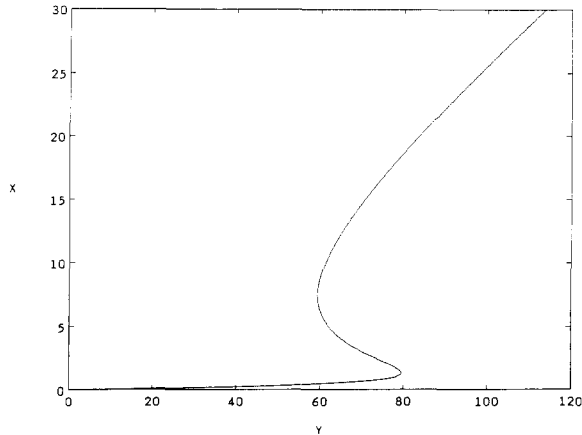


Fig. 6. Normalized output intensity ( $X$ ) versus normalized input intensity ( $Y$ ) for Kerr-like dispersive bistability, with  $T=0.1$ ,  $\alpha l=0.1$ ,  $\Gamma' l=2$  and  $\delta_0=0$ .

This is exactly the same state equation as in eq. (32) of ref. [2] for saturable absorptive O.B. in a ring cavity. The bistability condition, the inflection point and the extremes of the graph are all the same as in ref. [2]. In fig. 5, we take the square root of (22) and show how the exact solution approaches the limit

(24). Lines (a) to (e) are the exact solutions, with  $C=-20$ , and  $(\gamma_0, T) = (-1, 0.025)$ ,  $(-0.8, 0.02)$ ,  $(-0.6, 0.015)$ ,  $(-0.4, 0.01)$ ,  $(-0.2, 0.005)$  respectively. Line (f) is the mean field limit.

(ii) *Kerr-like photorefractive (PR) optical-bistability (O.B.)*

For the set of  $(E_0, \Delta)$  which render  $\gamma$  imaginary, eq. (19) becomes

$$Y = (X/T^2) \{ [1 - R \exp(-\alpha l)]^2 + 4R \exp(-\alpha l) \sin^2[\frac{1}{2}[\Gamma' l/(1+X) + \delta_0]] \}. \tag{25}$$

Fig. 6 displays Kerr-like dispersive bistability, with  $T=0.1$ ,  $\alpha=0.1$ ,  $\Gamma' l=2$ ,  $\delta_0=0$ . Multistability is possible when  $\sin^2[\frac{1}{2}[\Gamma' l/(1+X) + \delta_0]]$  undergoes several oscillations. The number of transitions depend only on  $\Gamma' l$ . Fig. 7 shows multistability for  $T=0.1$ ,  $\alpha=0.05$ ,  $\Gamma' l=40$  and  $\delta_0=0$ . For the sake of clarity, we enlarged the origin area in the inset.

In summary, our results expand the possible applications of the PR 2-BC scheme, as a very important building block in nonlinear optical. We showed a very close analogy between PR 2-BC and homo-

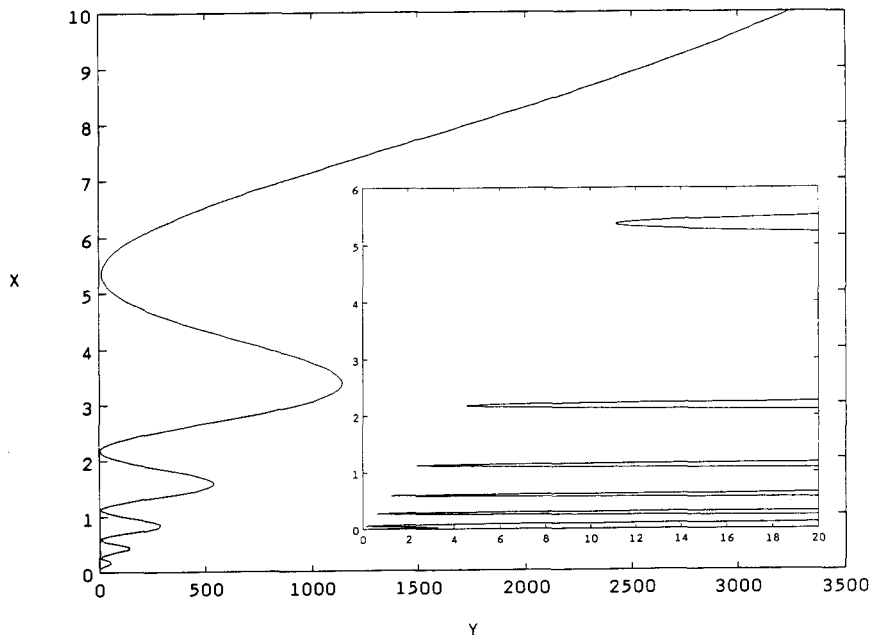


Fig. 7. Multistability in PR ring cavity. Here  $T=0.01$ ,  $\alpha l=0.05$ ,  $\Gamma' l=40$  and  $\delta_0=0$ . The oscillations near the origin are enlarged in the inset.

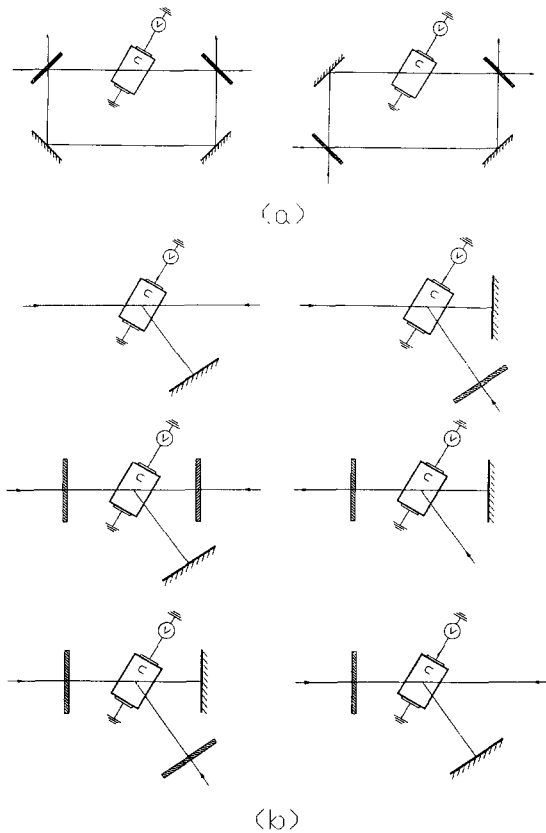


Fig. 8. (a) Double directional rings with intracavity two-beam coupling via reflection gratings. Inputs and outputs are designated with arrows. (b) Possible bistable four-wave mixing configurations.

generously broadened two-level atom system. Since PR materials exhibit large nonlinearities with moderate power and CW laser beams, these new applications may prove to be very attractive. The requirement of two beams in the basic building block may give new architectures, utilizing two-port input, two-port output devices. Arrays of Fabry-Perot etalons, with contradirectional input light beams are such possible switching devices.

We see from the numerical calculations that moderate coupling coefficients are enough to obtain strong O.B. The bottleneck of the PR effect is its speed.  $\text{Bi}_{12}\text{SiO}_{20}$  (BSO) and GaAs are both photorefractive, and have the fastest time response. GaAs has also the capability for integration, which makes it a preferable choice.

We have presented here only the simplest configuration. Two-beam coupling schemes, which reflection gratings (Fabry-Perot, ring) and four-wave mixing configurations may exhibit similar and even richer features. Figs. 8(a) and 8(b) shows some of these possible schemes. The dynamics of the processes involved is also of great interest and is currently under investigation.

This research was supported by the Foundation of Research in Electronics, Computers and Communications, Administrated by the Israel Academy of Science and Humanities, and a grant from the National Council for Research and Development, Israel, and the European Economic Community.

## References

- [1] Y.R. Shen, *The principles of nonlinear optics* (Wiley, New York, 1984).
- [2] L.A. Lugiato, in: *Progress in optics*, ed. E. Wolf (North-Holland, Amsterdam, 1984) pp. 71-216.
- [3] B. Fischer, M. Cronin-Golomb, J.O. White and A. Yariv, *Optics Lett.* 6 (1981) 519.
- [4] Y. Fainman, E. Klanchik and S.H. Lee, *Opt. Eng.* 25 (1986) 228, and references cited therein.
- [5] M. Cronin-Golomb, B. Fischer, J.O. White and A. Yariv, *IEEE J. Quantum Electron.* QE-20 (1984) 12.
- [6] N.V. Kukhtarev, V.B. Markov, S.G. Odulov, M.S. Soskin and V.L. Vinetskii, *Ferroelectrics* 22 (1979) 949.
- [7] B. Fischer, *Optics Lett.* 11 (1986) 236.
- [8] P. Yeh, *J. Opt. Soc. Am. B* 3 (1986) 747.
- [9] S.K. Kwong, M. Cronin-Golomb and A. Yariv, *A.P.L.*, 45 (1984) 1016.
- [10] S.K. Kwong and A. Yariv, *Optics Lett.* 11 (1986) 377.
- [11] R.W. Eason and A.M.C. Smont, *Optics Lett.* 12 (1987) 51.
- [12] J. Rodriguez, A. Siahmakoun and G. Salamo, *Appl. Optics* 26 (1987) 2263.
- [13] K.D. Shaw and M. Cronin-Golomb, *Optics Comm.* 65 (1988) 301.
- [14] J. Wang and J. Yao, in *Technical Digest, Topical Meeting on Photorefractive materials, effects and devices* (Optical Society of America, Washington, DC, 1987) p. 29.
- [15] R. Yahalom and A. Yariv, *J. Opt. Soc. Am. B* 5 (1988) 1783.
- [16] G.P. Agrawal and C. Flytzanis, *IEEE J. Quantum Electron.* QE-17 (1981) 374.
- [17] P. Yeh, *Optics Comm.* 45 (1983) 323.
- [18] R. Bonifacio and L.A. Lugiato, *Lett. Nuovo Cim.* 21 (1978) 505.
- [19] R. Bonifacio, M. Grouchi and L.A. Lugiato, *Nuovo Cim.* B53 (1979) 311.