Spatial light modulation and filtering effects in photorefractive wave mixing

Baruch Fischer, Shimon Weiss, and Shmuel Sternklar
Department of Electrical Engineering, Technion-Israel Institute of Technology, Haifa 32000, Israel
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A study of photorefractive wave mixing with image bearing beams reveals unique capabilities in spatial modulation, filtering, and cleanup of amplitude and phase modulated beams. We study and explain the negligible image crosstalk in various photorefractive oscillators.

In this letter we demonstrate and explain the lack of spatial information crosstalk between beams in various oscillators by photorefractive (PR) wave mixing (WM).1,2 This study is necessary to explain a basic unresolved problem concealed in the operation of passive (self-pumped) phase conjugate mirrors (PPCM's). We show that the situation here is basically different from conventional four-wave mixing (4 WM) with externally supplied pumps; there, crosstalk is inherent, although negligible in some cases3,4 (with nearly plane wave pumps), and is used, for example, for correlation and convolution applications.5 We also suggest the use of a particular 4WM configuration, the double phase conjugate mirror (DPCM) as a bidirectional spatial light modulator and controllable filter, and for achieving cleanup of amplitude and phase distorted beams. This was done before only in 2WM for phase distorted beams.6,7

Recently,8,9 we have described the operation of the DPCM, which is pumped from opposite sides of a photorefractive crystal, as shown in Fig. 1. Two beams with complex amplitudes $A_2$ and $A_4$, which may carry pictorial information, are coupled and self-refracted into each other. This results from a dynamic 4WM process in which self-induced phase gratings develop in the intersection region of the two input beams. In this process the two input beams are channeled and mutually bent into each other giving output beams 1 and 3 which carry the information of beams 2 and 4, respectively. The output beams were found to be phase conjugate of the input beams, with amplitudes $A_1 \propto A_2^*$ and $A_3 \propto A_4^*$. No image crosstalk was observed, i.e., images $A_1$ and $A_3$ do not affect beams $A_2$ and $A_4$, respectively. Thus the two spatially complex inputs totally exchange their spatial information as they are guided into a counter-propagating light channel through the crystal.

We carried out a detailed experimental study to further check the possible existence of crosstalk between the beams in the DPCM. In the setup shown in Fig. 1, transparencies $T_1$ and $T_2$ were Ronchi rulings of 4 lines/mm, oriented parallel and perpendicular, respectively, to the plane of the paper. The lenses $L_1$ and $L_2$ with focal length $f = 10$ cm imaged the gratings with an image reduction of 10:1 at an overlapping image plane in the PR BaTiO$_3$ crystal, where the beam diameter for both inputs was 2 mm. The crystal is and beam's geometry as well as the extraordinary polarized output of an argon ion laser is similar to the experiment described in Refs. 8 and 9. The phase conjugate output beams 3 and 1 were Fourier transformed and projected on screens where the pictures of Fig. 2 were taken. The Fourier transforms of the two orthogonally oriented gratings are clearly visible. Any crosstalk would be manifested as an orthogonal light distribution relative to the Fourier distribution; none is detected here.

Other imaging and light focusing configurations, as well as a similar check with either $T_1$ or $T_2$ removed while examining the intensity and phase profiles at the unmodulated beams side, lead to the same conclusion: no significant crosstalk exists in the DPCM.

Other PR oscillators exhibit similar features. The phase conjugating capability of the ring PPCM1 is based on the lack of crosstalk. Although the feedback pump through the ring configuration is carrying the image of the signal, it does not deteriorate the phase conjugation. In the semilinear PPCM the common belief attributed the phase conjugating capability to the cleaned-up self-generated pumps.1 Recent experiments involving oscillators with image bearing pumps10 in the two cascaded semilinear PPCM's rule out this explanation. The self-generated pumps were spatially modulated, but phase conjugation was still obtained. An image bearing oscillator with two facing DPCM's exhibited similar isolation features9 where the external input pumps for the DPCM's and the oscillation between them were carrying images.

Two-beam coupling in PR crystals might be different since the grating is written initially by externally supplied beams unlike the previous configurations. It was shown in the last6 that no phase crosstalk occurs in two-beam coupling. This is not surprising in light of a simple holographic description which will be discussed below. The negligible crosstalk of images with amplitude information is, however, of considerable interest. An experiment was carried out to check the amount of crosstalk in two-beam coupling. We set up the 2WM configuration shown in Fig. 3. Ronchi rulings with 0.8 lines/mm and perpendicular gratings' orientations were placed at $T_1$ and $T_2$. Lenses $L_1$ and $L_2$ projected a reduced image of 10:1, or their Fourier, or close to Fourier, transforms onto the crystal. Typical far-field patterns of the amplified output beam are shown in Fig. 4(a). We noticed that crosstalk exists compared to its nonsignificant effect in the DPCM. It is negligible only for very efficient two-beam coupling, especially for Fourier plane mixing. When we in-

FIG. 1. Double phase conjugate mirror (DPCM). $C$ is the photorefractive crystal, pumped by input beams 2 and 4, which are spatially modulated by slides $T_1$ and $T_2$. $L_1$ and $L_2$ are lenses. Beams 1 and 3 are self-generated.
tentionally reduced the efficiency by changing the angle of the crystal, or during the time until efficient coupling was built up, image crosstalk was observed. This crosstalk was significant in the direction perpendicular to the beam's plane [Fig. 4(b)]. These results will be discussed below.

From a simple holographic point of view, we expect image crosstalk of the interacting beams. Reading a hologram, written by beams $A_1$ and $A_4$, with one of the image bearing writing beams ($A_1'$), affects the diffracted beam of which the amplitude is proportional to $A_1' (A_1 A_4) = |A_1|^2 A_4$, where $A_1$ and $A_4$ are the beams' amplitudes and $x$ represents the transverse coordinates. A similar situation occurs when the reading beam is the phase conjugate of one of the writing beams, $A_2 \propto A_1^*$, which gives a diffracted beam $A_3 \propto A_2 (A_1 A_4^*) \propto |A_1|^2 A_4^*$. In photorefractive materials where the writing and the reading of the hologram occur simultaneously, the above two schemes represent nonlinear two- and four-wave mixing. Thus the image $A_1(x)$ is expected to affect the diffracted beam by a factor of $|A_1(x)|^2$. [Note that the phase of $A_1(x)$ is canceled out.] The experimental evidence shows, however, the absence of significant crosstalk in the photorefractive oscillators described above.

A helpful insight into the crosstalk mechanism may be gained by describing the image bearing beams as an integral of plane wave components with slightly different directions (by Fourier transformation). These components are represented by waves $A_1$ and $A_2$ for beam 2 of the DPCM in Fig. 1. Crosstalk arises where one plane-wave component (say $A_2$) is diffracted by the gratings written by another component ($A_1$ or its phase conjugate beam $A_1^*$), and vice versa. This cross diffraction might be negligible for particular image patterns. In our devices, however, a basic filtering mechanism eliminates such diffractions.

We suggest that the two following factors are responsible for the reduction of image crosstalk in PR oscillators. One is the strong volume gratings selectivity after oscillation builds up. The other factor is the preference of phase conjugate pairs which give highest PR gain due to the overlapping of two sets of gratings. In the 4WM scheme of Fig. 1 the sum of the amplitudes of the two sets of transmission gratings, induced by the beams couples ($A_1 A_4$) and ($A_2 A_3$), is proportional to $A_1 A_4^* + A_2 A_3^*$. If the counter-propagating beams are phase conjugate beams (i.e., $A_1 \propto A_4^*$ and $A_2 \propto A_3^*$) the terms overlap in space. It permits the mutual strengthening and buildup of the two sets of gratings and the self-generated beams, and gives the highest PR gain. This situation that optimizes the gratings' writing may still suffer from information crosstalk in the reading stage as described above. Since the distorted diffracted beam itself participates in writing the very same gratings, it can lead to an iterative walkoff from the phase conjugate solution and wash out of the gratings. The volume gratings selectivity in the PR oscillation restricts this walkoff as will be explained below.

The filtering effect is analyzed for the DPCM. We consider the simple scheme of Fig. 1 in which beam 2' (replaces 2) is misaligned with respect to beam 1, such that their wave vectors $k_1 + k_2 = \Delta k \neq 0$. This is still a 4WM configuration with a phase mismatch $\Delta k$, which will show the angular selectivity of the mixing process.

The coupled wave equations for beams 1 and 3 in the nondepleted pumps (beams 2' and 4) approximation for transmission gratings and negligible absorption are

$$
\frac{dA_1}{dz} = -\frac{\gamma}{I_0} \left[ |A_1|^2 A_4 + (A_1^* A_4) A_3 e^{i\Delta k z} \right],
$$

$$
\frac{dA_3}{dz} = \frac{\gamma}{I_0} \left[ |A_3|^2 A_4 + (A_4^* A_3) A_1 e^{i\Delta k z} \right],
$$

where the prime of $2'$ is omitted, $\gamma$ is the coupling constant, $I_0$ is the total light intensity, $I_0 \approx I_2 + I_4$, $I_1 = |A_1|^2$, and $\Delta k_z$ is the component of $\Delta k$ along the $z$ axis. We note that for $\Delta k = 0$, the full nonlinear problem is exactly solvable. A comparison with the present approximate method will be given below.

With the boundary conditions $A_1(z = 0) = A_1(0)$ and $A_3(z = l) = 0$, we obtain from Eqs. (1)
where $s = (1/2) \left[ p^2 \gamma^2 - 2 p (i \Delta k_z) - (\Delta k_z)^2 \right]^{1/2}$, $p = (1 - q)/(1 + q)$, and $q = I_d / I_s$. For $\Delta k = 0$, Eqs. (2) become

$$\rho = \frac{A_1}{A_0} \frac{e^{\gamma l} - 1}{e^{\gamma l} - q}, \quad t = \frac{e^{\gamma l} (1 - q)}{e^{\gamma l} - q}. \quad (3)$$

The infinities of $\rho$ and $t$ permit the self-buildup of beams 1 and 3 and give the operating points of the DPCM. Thus, our approximate method predicts the operation of the DPCM even for real $\gamma$ (for the common case in PR materials where the phase shift between the induced gratings and the fringes is $\pi/2$). The oscillation $(\rho = t = \infty)$ occurs at

$$\gamma l = (\ln q) (1 + q)/(1 - q) \quad (4)$$

for $\Delta k = 0$. The threshold value of the coupling constant is $|\gamma l|_{th} = 2$, for the pump ratio $q = 1$. Where $q$ approaches 0 or $\infty$, the needed $\gamma l$ increases to $\infty$. The agreement with the exact theory is striking. The only difference is the specific relation between $\gamma l$ and $q$ for oscillation. The exact theory, which has more flexibility in self-selecting the amount of pump depletion, predicts oscillation in a range of $q$ for any value of $\gamma l$ above threshold. Similar results and agreement can be derived for the ring PPCM, which is also described by Eqs. (2) with the appropriate boundary conditions.1

The results of Eqs. (2) for the DPCM resemble those of the distributed feedback (DFB) laser11 and the standard externally pumped 4WM with phase mismatch. 1,3 A basic difference is that in the latter cases oscillation is possible only for $\Delta k \neq 0$ or a nonreal coupling constant. In the DFB laser this dictates a detuning from the frequency that matches the Bragg condition. In the 4WM configuration this can be done by misaligned or frequency detuned beams, that affects $\Delta k$ and $\gamma$, or by applying electric field on the PR crystal (modifies the $\pi/2$ phase shift).4 In the DPCM, however, oscillation is achievable for $\Delta k = 0$. We also note that in all PR wave mixing configurations, but not in the DFB laser, the effective coupling parameters are dependent upon the pump intensity ratio.

Since small deviations of $\Delta k$ from zero cause a drastic decrease of $\rho$ and $t$ (see Fig. 2 of Ref. 3), oscillation is restricted to $\Delta k = 0$. Other solutions of Eq. (2) with nonzero $\Delta k$, as in the DFB laser, are not permissible here because of the deterioration of the effective gain which is optimized for phase conjugate beams.

The absence of image crosstalk in the DPCM is now clear. Since its operation is based on the oscillation and self-buildup of beams 1, 3, and the gratings, a complete phase matching and phase conjugation is needed. This dictates the elimination of cross diffractions. Similar arguments hold for other PR oscillators. Thus the ring and semilinear PPCM also involve crosstalk elimination and buildup of phase conjugate beams. In the two-beam coupling, which is a nonoscillatory device, crosstalk is observable (Fig. 4), especially in the direction perpendicular to the pumps' plane, in a conical pattern, where phase matching exists.12,13 Such a cone of diffracted light is observable in nonefficient operation of the DPCM and other PPCM's before collapsing to the phase conjugating mode. In other directions, the crosstalk is much reduced (Fig. 4) by the volume gratings filtering. A collapse to an image isolated state occurs for very efficient coupling and strong beam amplification, where the operation approaches a quasi-oscillatory behavior.

The use of the DPCM and 2WM for phase and amplitude distorted beam cleanup is clear. In a demonstration of this capability we obtained the cleaned-up wave pattern shown in Fig. 5. The obvious advantage of the DPCM over the 2WM configuration has already been pointed out.6 The DPCM can be operated with two beams from different lasers; thus suggesting filtering and spatial modulation capabilities of a "remote" distorted beam by a "local" laser source. When the two input beams are modulated, it acts as a bidirectional light modulator. Other optical processing applications, such as pictorial edge enhancement, thresholding, and implementations of iterative image buildup algorithms will be discussed elsewhere.

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11See, for example, A. Yariv, Quantum Electronics (Wiley, New York, 1975), Chap. 19.

![Fig. 5. Amplitude and phase distorted beam cleanup: (a) pump's image; (b) cleaned beam in 2WM mixing in image plane.](image-url)