New optical gyroscope based on the ring passive phase conjugator

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A new optical rotation sensor is described. It is a ring passive phase conjugator in which the ring may consist of a multimode fiber. A nonreciprocal phase shift in the "passive" like fiber ring activates a grating movement and subsequent frequency detuning of the beams in a photorefractive four-wave mixer. This device has the advantages of natural reciprocal behavior of phase conjugate beams (essential for rotation sensing) and has several adjustable controlling parameters. It reveals a new class of interferometry in which changes in the ring's optical phases, the beam's intensities and losses, and the mixing crystal's efficiency and electric field modulate a frequency detuning of the oscillating beams.

Current optical gyroscopes are divided into two categories[1]: fiber and laser gyros. In the passive fiber gyro, rotation causes a nonreciprocal optical path and phase difference to arise between the counterpropagating beams in the ring of a Sagnac Interferometer. Significant phase difference can only be achieved with a long fiber ring. Even then, the phase difference is very small. For this reason all other nonreciprocal phase sources must be identified and eliminated. This dictates, for example, the use of a single mode fiber at a single polarization to ensure complete reciprocity at rest. In the active laser gyroscope, the nonreciprocal optical length of a ring produces different frequencies for the counterpropagating beams in an active ring laser cavity. This gyro suffers as well from unwanted nonreciprocal contributions, in addition to frequency "lock-in" at low rotation rates.[2]

In this letter a new type of interferometer for rotation sensing is described: the ring passive phase conjugate gyroscope (ring PPC gyro). It is based on a ring passive phase conjugator,[3] in which self-oscillation by self-induced gratings in a photorefractive four-wave mixer produces the phase conjugate forms of the input beam 2 and probe beam 4 which are linked by the ring (Fig. 1).

It is shown that a nonreciprocal phase shift in the ring gives rise to a determined movement of the grating in the photorefractive material. This, in turn, results in a self-frequency detuning of the counterpropagating beams. It is an activelike gyro but the detuning originates from the mixing crystal activated by the nonreciprocity in the ring. Reasonable rotation sensitivity may be obtained by a long fiber ring as in a fiber gyro.The natural "time reversal" properties of phase conjugation eliminate much of the reciprocity problems of fiber gyros and allow the use of a multimode fiber for the ring. Since the device is based on self-pumped four-wave mixing, it does not impose a strict coherence requirement on the laser, and a variety of adjustable parameters is available for controlling and biasing the frequency shift of the device.

A scheme of the ring PPC gyro is shown in Fig. 1. The input beam 2 passes through the photorefractive crystal and is fed back by the fiber ring into the crystal as beam 4. The other two beams, 3 and 1, are linked by the same ring and build up as oscillations via the self-induced gratings in the crystal.

The basic theoretical study of this system in a degenerate case has been described in an earlier work.[3] However, the possibility of nondegenerate behavior in the passive four-wave mixer, which has been observed recently,[4] was not accounted for.

A detailed general analysis of nondegenerate[5] four-wave mixing shows that the nondegeneracy may be included in all the expressions of Ref. 3, by allowing the coupling constant γ to be complex. It will be shown that the crystal through a complex γ, provides for a self-adjusting phase lag of the grating with respect to the fringes. Thus, a complete general analysis necessitates dealing with phases and amplitudes rather than intensities of the beams. We shall make use of the exact solution that is summarized in Ref. 3 for transmission gratings, where the beams were taken to be plane waves and absorption was neglected.

The boundary conditions of the ring PPC for the complex amplitudes A_i of the beams at the faces $z = 0$ and $z = l$ of the photorefractive crystal are

$$A_2(z = l) = A_2(l), \tag{1}$$

$$\frac{A_1(0)}{A_3(0)} = m, \quad \frac{A_4(0)}{A_2(0)} = \tilde{m}, \tag{2}$$

where m and $\tilde{m}$ are the complex amplitude transmissivities of the same optical path for the two counterpropagating beams in the ring. Thus, $m = \tilde{m}$ unless a nonreciprocal phase of $\vartheta/2$ exists in the fiber linking the beams. Then, $m\tilde{m}^* = |m|^2 e^{i\vartheta} = Me^{i\vartheta}$.

The following equations are derived in a manner similar to that of Ref. 3:

$$m_1 = \frac{T + Q}{m_2[(\Delta + B)T + Q]}, \tag{3}$$

$$r = -\frac{(\Delta + 1)T}{m_2[\Delta T + Q]}, \tag{4}$$

where $r = A_3(0)/A_4^*(0)$, $m_1 = A_1(0)/A_2^*(0)$, $m_2 = A_2^*(l)/$

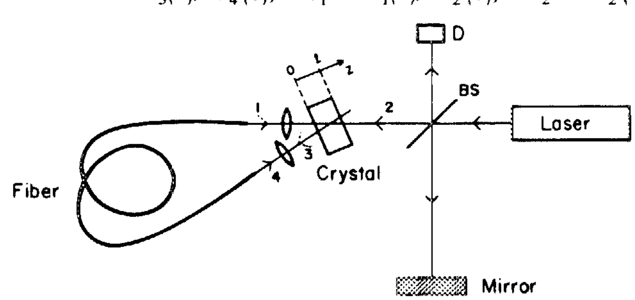

FIG. 1. Ring PPC gyro. Light is directed via beam splitter BS to the reference arm which may have a simple mirror or another multimode fiber and a linear PPC.

$A_1(l)$, $R = 1/|m_2|^2$ is the reflectivity of the device, $Q = [\Delta^2 + (\Delta + 1)^2 R]^{1/2}$, $T = \tanh[(\gamma l/2)Q]$, $B = (1 + \Delta)R$, $\Delta = (1 - M)/(1 + M)$ is the normalized light power flux in the crystal, l is the width of the crystal, and γ is the coupling constant that depends on several parameters of the mixing crystal.[3]

The possibility of nondegenerate frequencies in our device means that the connected oscillating beams A_3 and A_1 are detuned by δ with respect to the given frequency of the input A_2 and A_4. The dependence of γ on the detuned frequency arises because of the finite response time of the gratings built up in the crystal.[6]

$$\gamma(\delta) = \gamma_0/[1 + i(\tau\delta)], \tag{5}$$

where τ is the response time that depends on several parameters. For a given crystal its dependence on the total beam power is important. It is approximately scaled inversely with power density. For $BaTiO_3$, $\tau \sim (10/I)$ seconds,[3] where I has units mW/mm^2.

The detuned frequency and the reflectivity of the device can be found by noting that

$$m_1/r = m\bar{m}^* = Me^{i\vartheta}, \tag{6}$$

where ϑ is any nonreciprocal phase in the ring, M is the ring's transmittance, and comparing it to the ratio given by Eqs. (3) and (4),

$$\frac{(T + Q)(\Delta T + Q)}{[(\Delta + B)T + Q](\Delta + 1)T} = -Me^{i\vartheta}. \tag{7}$$

The equation is solved numerically for the reflectivity R and the normalized self-detuning $(\tau\delta)$ as functions of ϑ. Typical plots are shown in Fig. 2(a). Our main interest here is in the $\delta(\vartheta)$ behavior, which is the measurable quantity of the nonreciprocal phase, or the rotation rate. Fortunately, around $\vartheta = 0$ its behavior is nearly linear. In Figs. 2(b) and 2(c) the linear region is enlarged and plots of $(\tau\delta)$ are shown for $\gamma_0 l = -4$ and varying M, and for $M = 0.75$ and varying $\gamma_0 l$.

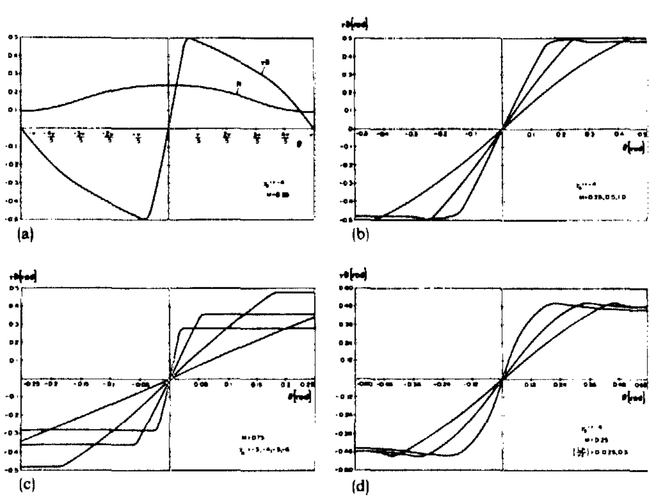

FIG. 2. (a) Reflectivity and detuning as a function of the nonreciprocal phase shift ϑ of the ring PPC gyro. (b) Detuning in the linear region for $M = 0.25, 0.5, 1$. Larger slopes correspond to larger M. (c) Detuning in the linear region for $(\gamma_0 l) = -3, -4, -5, -6$. Larger slopes correspond to larger $(\gamma_0 l)$. (d) Detuning in the linear region for $Ln/(\tau c) = 0, 0.25, 0.5$. Larger slopes correspond to larger $Ln/(\tau c)$.

The slope and sensitivity are larger for larger M (less loss in the ring) and larger absolute values of $\gamma_0 l$ (greater coupling in the crystal). The reflectivity R is almost constant, $R \approx M$ in this region. In all of the graphs we observe sharp transition of the slope at the edges of the linear region.

An expression for $\tau\delta(\vartheta)$ in the linear region can be easily derived from Eq. (7):

$$(\tau\delta) \approx \alpha\vartheta, \tag{8}$$

where

$$\alpha \simeq \frac{\tanh(\gamma_0 l/2)}{(\gamma_0 l/2)[1 - \tanh^2(\gamma_0 l/2)]} \frac{M}{M + 1}.$$

We approximated $R \approx M$, valid for large $(\gamma_0 l)$ (which has a minimum threshold value of 1 at $M = 1$ and increases for lower M).[3]

Using the ring PPC as a gyroscope for different regimes of rotation rates depends on the crystal time constant τ and the ring area (or equivalently its diameter and total length). Significant nonreciprocal phase shifts due to rotation may be obtained by long fibers as in fiber gyros. The ϑ dependence on the rotation rate Ω for a ring consisting of a coil of fibers of total length L and ring diameter D is given by[1]

$$\vartheta = (2\pi LD/\lambda_0 c_0)\Omega, \tag{9}$$

where λ_0 and c_0 are the wavelength and velocity of light in vacuum. Thus we obtain

$$\delta = \frac{\alpha}{\tau}\left(\frac{2\pi LD}{\lambda_0 c_0}\right)\Omega. \tag{10}$$

For typical values of $\lambda_0 = 0.5\ \mu m$, $D = 0.25$ m, we have $\delta \approx 10^{-2} \times (\alpha L/\tau)\Omega$. Since $\alpha \sim 1$, in order to measure small rotation rates on the order of 10^{-6} (rad/s) ($\sim 10^{-2}$ times earth's rotation rate, a requirement in navigational applications) with detuning of $\delta \sim 10$ rad/s, we must have $(L/\tau) \sim 10^9$. Thus for a fiber length of 10^3 m, $\tau \sim 10^{-6}$ s. Small time constants of this order of magnitude may be achievable, for example, in photorefractive semiconductors.[7,8] When the coupling efficiency of these semiconductors will improve, integration with semiconductor lasers will be an attractive option. We emphasize here the possibility of tuning the τ of a specific photorefractive material by the laser light power density, as was mentioned earlier.

In cases where long fiber lengths are needed for the ring, another factor may be of importance, namely, the cavity-dependent asymmetry due to the detuning: $\vartheta' = L\delta/c$. Here $c = c_0/n$, where n is the index of refraction of the fiber. Comparing ϑ' to ϑ of Eq. (9) shows that when $\delta \gtrsim \delta_0 = (\pi D/\lambda_0 n)\Omega$, ϑ' is significant. For $D = 0.25$ m, $\lambda_0 = 0.5\ \mu m$, and $n = 1.5$, $\delta_0 \sim 10^6\ \Omega$. Therefore, only for very small rotation rates on the order of $\Omega \sim 10^{-6}$ rad/s, must ϑ be renormalized by adding ϑ' to it. Then, in Eq. (7), we must substitute

$$\vartheta \to \vartheta + \vartheta' = \vartheta + [Ln/\tau c](\tau\delta). \tag{11}$$

Solving Eq. (7) with the renormalized ϑ results in the plots of $\tau\delta(\vartheta)$ shown in Fig. 2(d) for values of $(Ln/\tau c)$ in the high (L/τ) region $(Ln/\tau c = 0.25, 0.5)$. The effect of the added factor is an enhancement of the detuning in the linear region. For values of $(Ln/\tau c)$ beyond 0.7 this characteristic behavior of $\tau\delta(\vartheta)$ changes.

An attractive feature of the ring PPC gyro is its built-in

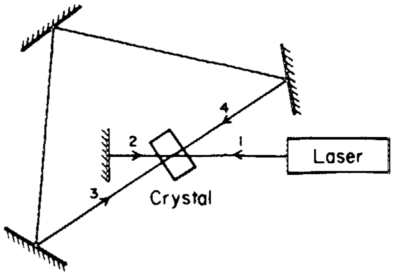

FIG. 3. Double directional ring oscillator with a photorefractive crystal.

reciprocity. Besides the frequency detuning effect, the self-induced grating adjusts itself in a manner which ensures spatial phase conjugation of the counterpropagating beams. Thus it is possible to use a multimode fiber as the ring cavity.

The detection of the detuning is done by interfering the reflection from the ring PPC with the input beam at a screen or detector D as shown in Fig. 1, as in a Michelson interferometer with a linear PPC described recently.[9]

In an experiment which we carried out, we set up a ring PPC with a photorefractive $BaTiO_3$ crystal[3] and a 1-m multimode fiber ring. The 488-nm line of an argon ion laser with horizontal polarization (extraordinary polarization in the crystal) was used as the input beam. The oscillation built up easily with proper coupling to the fiber, both with and without an étalon in the Ar^+ laser cavity. The polarization of the oscillation is maintained horizontal due to the much higher coupling efficiency for this component in $BaTiO_3$.[3] We induced a nonreciprocal phase in the ring via the Faraday effect. A magnetic field B of an electromagnet that was applied along a length L' of the fiber produced a nonreciprocal phase (for the horizontal polarization) $\vartheta = 2VBL'$, on the order of 0.1 rad, where V is the Verdet constant. Since $(\tau\delta) \sim \vartheta$ and $\tau \sim 1$ s, $\delta \approx 0.1$ rad/s. We indeed saw fringe movement of similar frequency. These fringes at D in Fig. 1 were of good quality as was reported for the linear PPC fiber interferometer.[9] Lower frequency detuning was observed even with zero applied magnetic field.[4] This can be explained by some residual nonreciprocity or an electric field (photovoltaic) in the crystal that shifts the zero in Fig. 2. A detailed analysis will be published elsewhere.

A variety of adjustable parameters is available in our device for controlling and biasing. The time constant τ that depends on the beam power, the transmittance of the ring M, and the magnitude of γ that depends on the orientation of the crystal can serve to change the sensitivity as well as the linear range of the device. Biasing the device may be done with an applied electric field on the crystal.[5] It acts to change the complex coupling constant[3] γ and shifts the zero of ϑ in Fig. 2.

A different configuration, the uni or double directional ring oscillator[10] shown in Fig. 3, may also serve as a gyroscope.[11] Its mechanism was misinterpreted[11] as a conventional laser ring oscillator where the only role of the photorefractive mixer was in producing the oscillating phase conjugate beams. In fact, this ring will have similar dynamics and detuning properties that depend on the reciprocal or nonreciprocal optical phases of the ring. A detailed study of this device will be presented elsewhere. One disadvantage is that the coherence of the laser beam must exceed the length of the ring to allow four-wave mixing in the crystal. In the ring PPC, however, coherence is not a problem in the mixing process since the writing beams produce their own writing mates or have the same optical path in the ring.[3] Nor does the interferometric measurement scheme between the reference beam and reflected beam 1 impose a constraint on the laser's coherence length. The ring length may be matched by a multimode fiber plus a linear PPC in the reference arm.[9]

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