Passive phase conjugate mirror based on self-induced oscillation in an optical ring cavity

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A passive phase conjugate mirror based on four-wave mixing in an optical ring cavity is described. Unlike previously demonstrated passive phase conjugate mirrors it generates only one of its pumping beams by nonlinear optical interactions, the other being provided by feedback of the probe after transmission through the nonlinear medium. The results of a theory yielding phase conjugate reflectivity and oscillation thresholds are presented together with an experimental demonstration of phase conjugation in barium titanate and strontium barium niobate. The device is self-starting by four-wave mixing, and has an oscillation threshold lower than that of other previously demonstrated passive phase conjugate mirrors with similar ease of alignment. The operation of a device which generates nonconjugate oscillation beams is also reported.

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The high efficiency of four-wave mixing in several photorefractive crystals such as barium titanate$^1$ and strontium barium niobate$^2$ makes them attractive candidates for phase conjugation down to very low light power levels.$^3$ In addition, the recently demonstrated use of these materials in passive phase conjugate mirrors$^3$–$^5$ has eliminated the necessity of using an external laser to provide high quality pumping beams coherent with the signal beam.

In the first demonstration of passive phase conjugation, the signal beam was allowed to pass through a photorefractive crystal in a linear cavity formed by two external mirrors [Fig. 1(a)]. Both pumping beams then built up in this cavity via nonlinear interactions fed by the signal beam itself. Phase conjugate reflectivities as high as 30% have been achieved so far, without techniques such as the use of antireflection coatings to reduce reflection losses. A theory of this device and its application in imaging experiments and as the end mirror of an argon ion laser have been recently reported.$^3$–$^5$ Henceforth, we identify this type of passive phase conjugate mirror as the linear mirror. Another passive phase conjugate mirror has also been reported which contains the first device and makes use of totally internally reflecting surfaces of the crystal as feedback mirrors.$^6$ We identify this device as the two interaction region (2IR) mirror, since it uses two linked interaction regions.

In this letter we introduce a new kind of single interaction region passive phase conjugate mirror: unlike the linear and 2IR mirrors it generates only one of its pumping beams via nonlinear optical interactions. The results of a theoretical analysis of this device are shown as well as experimental verification of its action as a phase conjugate mirror.

In the basic implementation of the new device [Fig. 1(b)] the signal beam 2 passes through a photorefractive medium and returns to it as pumping beam 4 around an optical ring cavity, here represented by mirrors $M_1$ and $M_2$. It may be advantageous to use curved mirrors or intracavity lenses to minimize diffractive loss of any spatial information on the signal beam. The possibility then arises that the nonlinear optical coupling in the crystal may be such that both the second pumping beam 3 and the phase conjugate beam 1 build up as oscillation beams in the ring cavity. We now turn to a theoretical examination of this possibility. As in the case of the linear mirror$^5$ use of the undepleted pumps approximation is inappropriate so we use the analysis of Ref. 7, where we derived the reflectivity of a holographic phase conjugate mirror allowing for pump depletion. After some further development of the theory to take account of the boundary conditions of the new device (henceforth named the ring mirror), we find that buildup of oscillation in the ring is in fact possible when the spatial phase shift between the holographic refractive index grating and the light interference pattern is nonzero so that advantage is taken of unidirectional beam coupling effects typical of real time holo-

![Fig. 1. (a) Geometry of the linear passive phase conjugate mirror. The probe is beam 4 and its phase conjugate is beam 3. The two pumping beams, 1 and 2, oscillate in the cavity bounded by mirrors $M_1$ and $M_2$. (b) Geometry of the ring passive phase conjugate mirror. Consistently with the convention that the intensity of beam 3 should be zero at its entrance face, the probe is designated as beam 2. Pump beam 4 is provided by reflection of the probe beam by mirrors $M_1$ and $M_2$. The second pump, beam 3 and the phase conjugate beam 1 are self-induced by the nonlinear medium. (c) Geometry of a four-wave mixing oscillator which can produce output beams which are not phase conjugates of the input beams (see text). Beams 2 and 4 are provided from an external source and beams 1 and 3 build up via four-wave mixing.](image-url)
Let the product of the intensity reflectivities of the feedback mirrors $M_1$ and $M_2$ be $M$ so that the appropriate boundary conditions for this device [Fig. 1(b)] are $I_j(0)/I_j(0) = M$ and $I_j(0)/I_j(0) = M$, where $I_j$ is the intensity of beam $j$. These boundary conditions are then used to find the constants of integration in the expressions derived in Ref. 7 for $A_j(z)/A_j(z)$ and $A_j(z)/A_j(z)$, where $A_j$ is the amplitude of beam $j$. The reflectivity, given by $|A_j(z)/A_j(z)|^2$, is shown in Fig. 2 as a function of $y/L$, a coupling constant characteristic of the medium. It has been taken to be real so that it represents the $\pi/2$ phase shift characteristic of holographic recording in photorefractive crystals by diffusion of charge carriers. Curves for several values of the feedback parameter $M$ are included.

The threshold coupling strength for $\pi/2$ phase shift is given by

$$y/L = \frac{(M + 1)}{(M - 1)} \ln \left( \frac{M + 1}{2M} \right).$$

(1)

This is a self-starting threshold; when it is exceeded, oscillation beams of infinitesimal intensity experience gain. This self-starting ability is not possessed by certain previously demonstrated passive phase conjugate mirrors, namely, linear mirrors in which the reflectivity of one of the cavity mirrors, $M$, is zero.\(^4\,5\) (semilinear mirror) the 2IR mirror.\(^6\) The oscillation beams of these devices experience gain only when their intensities are above a certain nonzero threshold. Starting these devices thus requires seeding of their oscillation beams. The 2IR mirror has in fact been shown to be able to start without the aid of externally provided seeding but this is believed to be dependent on effective seeding by the fanning effect\(^9\) which is due to two-beam coupling amplification of scattered light and not to the four-wave mixing process referred to here.

Moreover, Eq. (1) shows that the threshold coupling strength for unity feedback, $M = 1$, is $y/L = -1$. In Ref. 6 the self-starting threshold for the linear mirror was shown to be $y/L = 1/2$, where $M$ is the product of the intensity reflectivities of the linear cavity mirrors, so that with ideal feedback ($M = 1$) the threshold coupling strength is zero. The threshold of the semilinear mirror with unity reflectivity for the surviving external mirror was found to be $y/L = -2.49$, while the threshold for the 2IR mirror with ideal feedback\(^6\,10\) was $y/L = -4.68$.

The linear mirror thus has the simultaneous advantages of having very low threshold and of self-starting by four-wave mixing. It does, however, require careful alignment. Of the remaining passive phase conjugate mirrors, all of them easily aligned, the ring mirror seems to be most attractive in that it has the lowest threshold and is self-starting. A major advantage of the 2IR mirror has been that it is completely self-contained in a single crystal, using total internal reflection at the crystal faces for feedback. In the future, of course, the ring mirror could be implemented in the same manner using crystals whose surfaces are cut at appropriate angles.

The faithfulness of phase conjugation in these devices is a separate issue; it might be expected that since the self-induced pumping beams are not plane waves a certain amount of distortion would be introduced into the phase conjugate beam. In the case of the linear mirror, physical constraints imposed by the cavity mirrors may lead to filtering of the signal information from the pumping beams, but one of the pumps in the ring mirror is simply light transmitted through the crystal fed back to it by a passive optical system containing, at least in the experiment described below, very little spatial filtering. One plausible theory is that oscillation beams building up in these devices contribute most to the four-wave mixing process and experience maximum gain when the spatial overlap of counterpropagating beams is maximum, that is, when they are phase conjugates of each other. However, this explanation is inconsistent with the behavior of another device [Fig. 1(c)] which we built with the hope that it would be a phase conjugate mirror. A signal beam was incident on a beamsplitter and the transmitted beam was directed by mirror $M_1$ onto a barium titanate crystal as beam 2. The reflection from the beamsplitter was directed by mirror $M_2$ onto the crystal as beam 4. Theory indicates that with sufficient coupling strength and $\pi/2$ phase shift between the interference fringes and refractive index fringes, oscillation beams 1 and 3 can be expected to build up via gratings written between beams 1 and 4 and beams 2 and 3. The wave vector of the beam 1–beam 4 grating must be the same as that of the beam 2–beam 3 grating so that both combine to form a single grating coupling all four beams. Thus, we require

$$k_1 - k_4 = k_3 - k_2,$$

(2)

where $k_j$ is the wave vector of beam $j$. In conventional applications of four-wave mixing $k_1$, $k_2$, and $k_4$ are fixed so that Eq. (2) gives a unique value for $k_3$, the wave vector of the phase conjugate beam. Since only $k_1$ and $k_4$ are fixed in the device of Fig. 1(c), however, there is an extra degree of freedom manifested in the expected and observed appearance of beams 1 and 3 as cones of light with axis $k_3 - k_2$ and surfaces including both vectors $k_2$ and $k_4$. This means that self-induced oscillation by four-wave mixing does not always require that the counterpropagating beams be phase conjugates of each other.

The linear and ring mirrors also involve feedback to the crystal of the oscillation beams. In the linear mirror, beam 1 is reflected into beam 2 and vice versa by external mirrors, and in the ring mirror, beam 3 is reflected by external mir-
FIG. 3. Experimental arrangement used to demonstrate phase conjugation in the ring passive phase conjugate mirror. An argon ion laser was used at 488 nm in single longitudinal mode. Using a Cartesian coordinate system with the abscissa coincident with the incident beam direction, the locations of the elements measured in centimeters were 30 x objective $L_1$ (61.0), transparency $T$ (30.0), beamsplitter for observing phase conjugate reflection BS (50.0), 14 cm focal length 3-cm-diam lens $L_2$ (41.0), barium titinate crystal (0.0), plane mirror $M_1$ (9.0), plane mirror $M_2$ (28.0), 15 cm focal length 3-cm-diam lens $L_3$ (18.0). The c-axis of the crystal pointed in the direction of the vector (0.68, 0.73).

FIG. 4. Experimental results showing (a) the phase conjugate image of an Air Force Resolution Chart, (b) the phase conjugate image of uniform expanded beam vignetted by lens $L_2$, (c) the effect of fanning on the signal beam with apparatus as in Fig. 3, without transparency $T$ and mirrors $M_1$ and $M_2$ (beam transmitted through crystal before buildup of fanning), and (d) beam transmitted through crystal after buildup of fanning.

Experiments similar to those reported above have also been performed using a crystal of strontium barium niobate with a coupling strength somewhat smaller than that of the barium titinate. These were facilitated by the above mentioned advantages of the ring mirror: ease of alignment and low threshold.

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14. This coupling strength depends on parameters such as crystal orientation, electro-optic tensor, and index grating wave vector (see Ref. 11). When it is real, the phase shift between the light interference pattern and the refractive index grating is $\pi/2$. This is a behavior typical of photorefractive materials with no bias field. Where $y'$ is purely imaginary, the index grating is in phase with the light interference pattern, which is the case in media which have a local response such as atomic vapors.
16. We have doubled the value quoted in Ref. 6 to rescale it for comparison with our devices.
18. This crystal measured 5.1 x 4.8 x 5.1 mm and was poled into a single domain so that the c-axis was parallel to the 4.8-mm side.